Consistent perturbative/non-perturbative matching in heavy–quark production

Einan Gardi

based on:

M. Cacciari and E.G, *Nucl. Phys.* **B664** (2003) 299;
work by M. Cacciari and P. Nason;
and yet unpublished work with G.P. Korchemsky

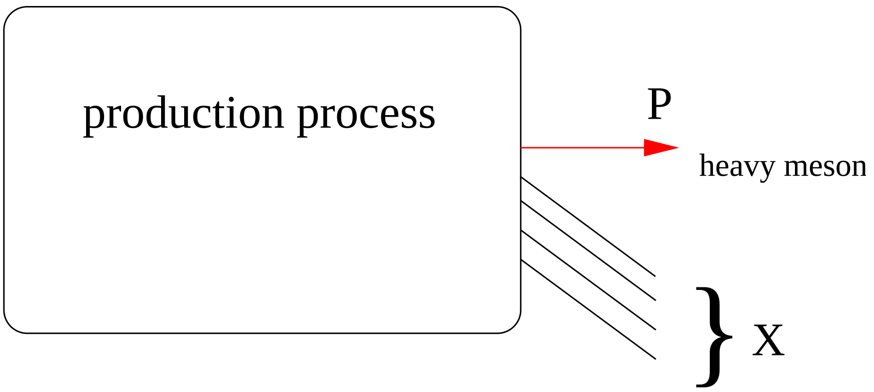
plan of the talk:

- Introduction: heavy–quark fragmentation function
- Motivation – Is there excess in B production at the Tevatron?
- Factorization; Renormalons and their cancellation
- Results of Dressed Gluon Exponentiation (DGE)

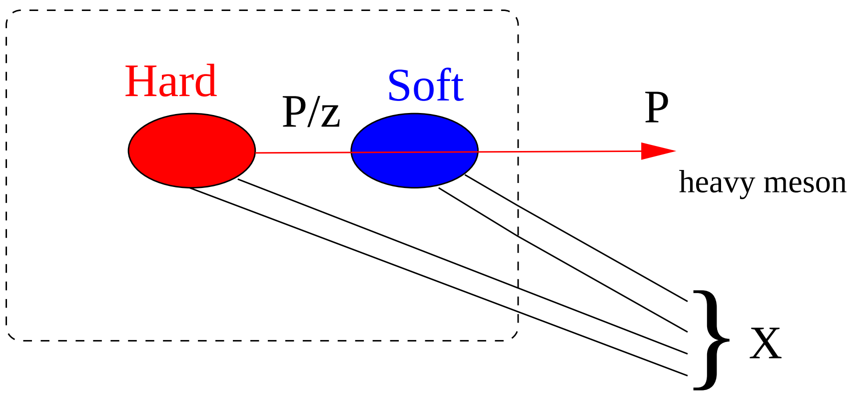
Heavy–quark production

Heavy-quark Fragmentation function $D(z)$

the probability to produce a heavy meson
with momentum fraction z of a heavy quark



factorization



$$d\sigma(x,Q^2)/dx = \int_x^1 C(x/z,Q^2;\mu^2)\, D(z;\mu^2)\, dz/z$$

C is the process–dependent coefficient function

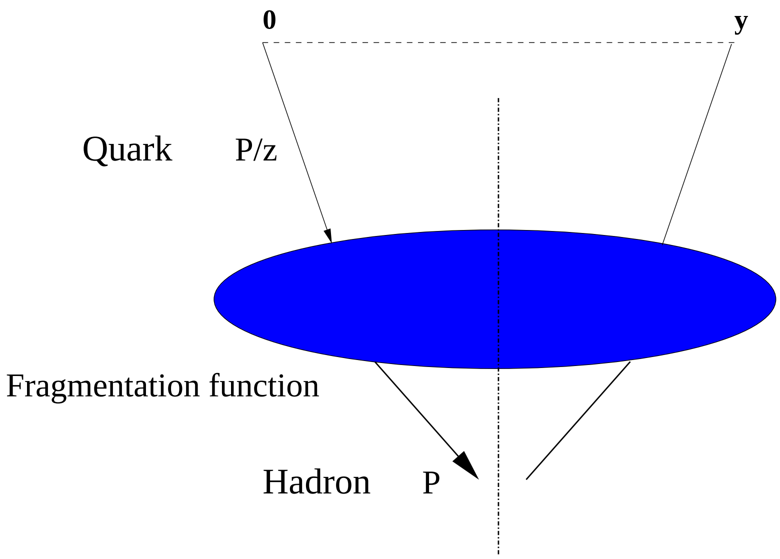
D is a universal fragmentation function

Example: $e^+e^-(Q) \longrightarrow B(E)+X$: measure $x \equiv 2E/Q$ distribution.

A perturbative approach to heavy-quark fragmentation?

$D(z;\mu^2)$ is a non-perturbative object: $b \longrightarrow B$

Defined as a matrix element with light-like separation y



can be computed replacing the heavy meson by an on-shell heavy quark:

It is infrared and collinear safe

D is inclusive
(does not depend on the details of the heavy-meson wave function)

real $\longleftrightarrow$ virtual cancellation of soft singularities

$m \gg \Lambda$: m is a cutoff for collinear radiation
(contrary to light-quark fragmentation)

Different aspects of factorization

Factorization into

- process dependent C and process independent D
- Hard C and soft D

through a convolution

$$d\sigma(x,Q^2)/dx = \int_x^1 C(x/z,Q^2;\mu^2)\, D(z;\mu^2)\, dz/z$$

or, a product in moment space $\left[\tilde{f}(N) \equiv \int_0^1 dz\; z^{N-1} f(z)\right]$

$$\widetilde{\sigma}(N,Q^2) = \widetilde{C}(N,Q^2;\mu^2) \times \widetilde{D}(N;\mu^2)$$

For light–quark fragmentation (as for DIS structure functions) we know only the μ^2 (logarithmic evolution) of D.
We have to compute C and parametrize D.

For heavy quarks we can compute D_{PT}.
It differs from the full D by *power corrections* (D_{NP}):

$$\widetilde{D}(N;\mu^2) = \widetilde{D}_{\text{PT}}(N,m^2;\mu^2) \times \widetilde{D}_{\text{NP}}(N,m^2)$$

It seems that $\widetilde{D}_{\text{NP}}(N,m^2)$ is suppressed by $\mathcal{O}(\Lambda/m)$.
If so the PT result (for B) would be within 5% of the full result.
BUT at large N the leading corrections are $\mathcal{O}(N\Lambda/m)$!!!

Asymptotic behaviour

For very large m: $D(z;\mu^2 \simeq m^2) \simeq \delta(1-z)$ (here the confinement effect is neglected)

A scaling law for $m \gg \Lambda$ and $z \longrightarrow 1$: $D(z;\mu^2 \simeq m^2) \simeq \frac{m}{\Lambda} f\left(\frac{m(1-z)}{\Lambda}\right)$

Taking moments: $\tilde{D}(N;\mu^2) = \int_0^1 dz\, z^{N-1} D(z;\mu^2)$

For $N \longrightarrow \infty$, $m \longrightarrow \infty$ with fixed m/N: $\tilde{D}(N;\mu^2 \simeq m^2) = \mathcal{F}(N\Lambda/m) + \mathcal{O}(1/N)$

The 'dead–cone'

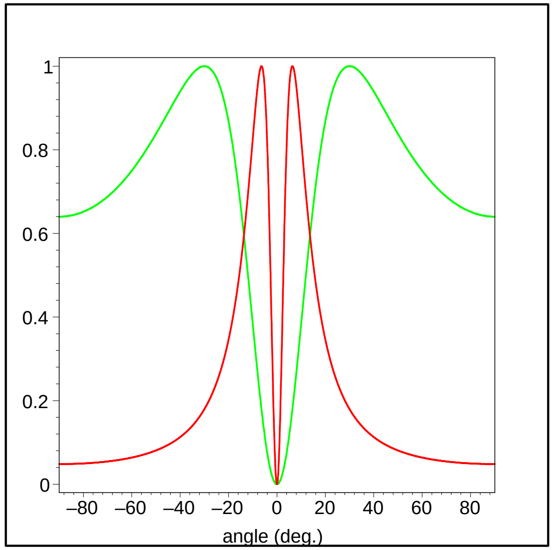
In a frame where $E \gg m$ the emission angle is $\sin^2\theta = \frac{k_\perp^2}{E^2 z^2 (1-z)^2}$

$$\frac{dD}{dz\, d\sin^2\theta} \simeq \frac{C_F\,\alpha_s}{\pi} \frac{1}{1-z} \frac{\sin^2\theta}{(\sin^2\theta + m^2/E^2)^2}$$

The radiation peaks at

$\theta \simeq m/E$

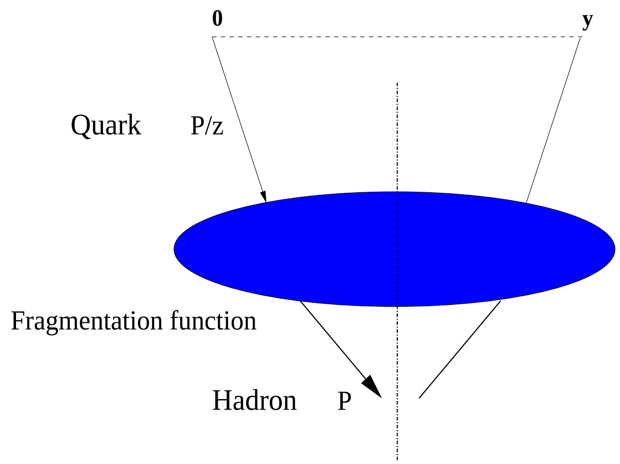
$|k_\perp| \simeq m(1-z)$



Asymptotic behaviour and large light-cone separations

$$F(py;\mu^2) \equiv \frac{1}{4\,N_c}\sum_X \mathrm{Tr}\left\{\langle 0|\not{y}\Psi(y)|H(p)+X\rangle\langle H(p)+X|\overline{\Psi}(0)|0\rangle_{\mu^2}\right\},$$

$$D(z;\mu^2) \equiv \frac{1}{2\pi\,z}\int_{-\infty}^{\infty}\frac{dy_-}{y_-}\exp(ipy/z)\,F(py;\mu^2),$$



Heavy-quark effective theory (large m): (Jaffe & Randall)

$$\frac{F(py,m^2)}{py}\,\exp(ipy) \longrightarrow \mathcal{F}(py\,\bar{\Lambda}/m) + \mathcal{O}(\bar{\Lambda}/m)$$

From the definition, at large N: (Cacciari & Gardi)

$$\tilde{D}(N;\mu^2\simeq m^2) \longrightarrow \left.\frac{F(py,m^2)}{py}\,\exp(ipy)\right|_{py=-iN} +\ \mathcal{O}(1/N)$$

Formally establishing the asymptotic scaling in the simultaneous limit:

$$\tilde{D}(N;\mu^2\simeq m^2) \simeq\ \mathcal{F}(py\,\bar{\Lambda}/m)\,\Big|_{py=-iN} + \mathcal{O}(1/N)$$

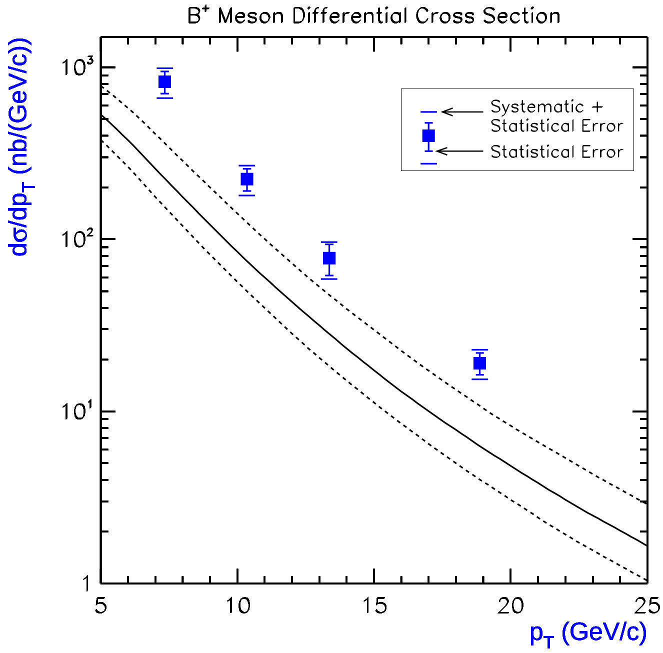
Heavy–quark fragmentation function

- Heavy–quark production is expressed as a convolution with a universal non-perturbative function $D(z;\mu^2)$
- In contrast with the light–quark fragmentation function the heavy–quark one $D(z;\mu^2)$ can be computed perturbatively.
- Its asymptotic behaviour is $D(z;\mu^2 \simeq m^2) = \frac{m}{\bar{\Lambda}} f\left(\frac{(1-z)m}{\bar{\Lambda}}\right)$

Motivation

CDF measurement

of B^+ production
in $p\bar{p}$ collisions:



$$\frac{d\sigma^B}{dp_T} = \frac{d\sigma^b}{dp_T} \otimes D^{b\to B}(z)$$

NLO, $D^{b\to B}(z) = \text{Peterson et al. form}, \ \epsilon = 0.006$

Data/Theory $= 2.9 \pm 0.5$ (exp.)

Cacciari & Nason: NLL resummation

Non-perturbative fragmentation function parameters determined from moments space e^+e^- data

Data/Theory $\simeq$ 1.7 $\pm$ 0.3 (exp.) $\pm$ 0.4 (th.)

Non-perturbative fragmentation parameters are very sensitive to the perturbative description

Significance of the large-z region — advantage to moment–space

Relevant NP information for B–meson hadro-production

Usually only specific features of the fragmentation–function spectrum are important.

Mellin moments definition:

$$\widetilde{D}^{b\to B}(N) \equiv \int_0^1 dz\; z^{N-1} D^{b\to B}(z)$$

The characteristic behaviour of the hadronic cross section is $\frac{d\sigma^b(p_T)}{dp_T} \simeq \frac{A}{p_T^5}$.

Therefore,

$$\begin{aligned}\frac{d\sigma^B(p_T)}{dp_T} &= \frac{d\sigma^b(p_T)}{dp_T} \otimes D^{b\to B}(z) \equiv \int d\hat{p}_T \int dz\, D^{b\to B}(z)\, \frac{d\sigma^b(\hat{p}_T)}{d\hat{p}_T}\, \delta(p_T - z\hat{p}_T) \\ &\simeq \frac{A}{p_T^5} \int dz\; z^4\; D^{b\to B}(z) = \frac{d\sigma^b(p_T)}{dp_T} \times \widetilde{D}^{b\to B}(N=5)\end{aligned}$$

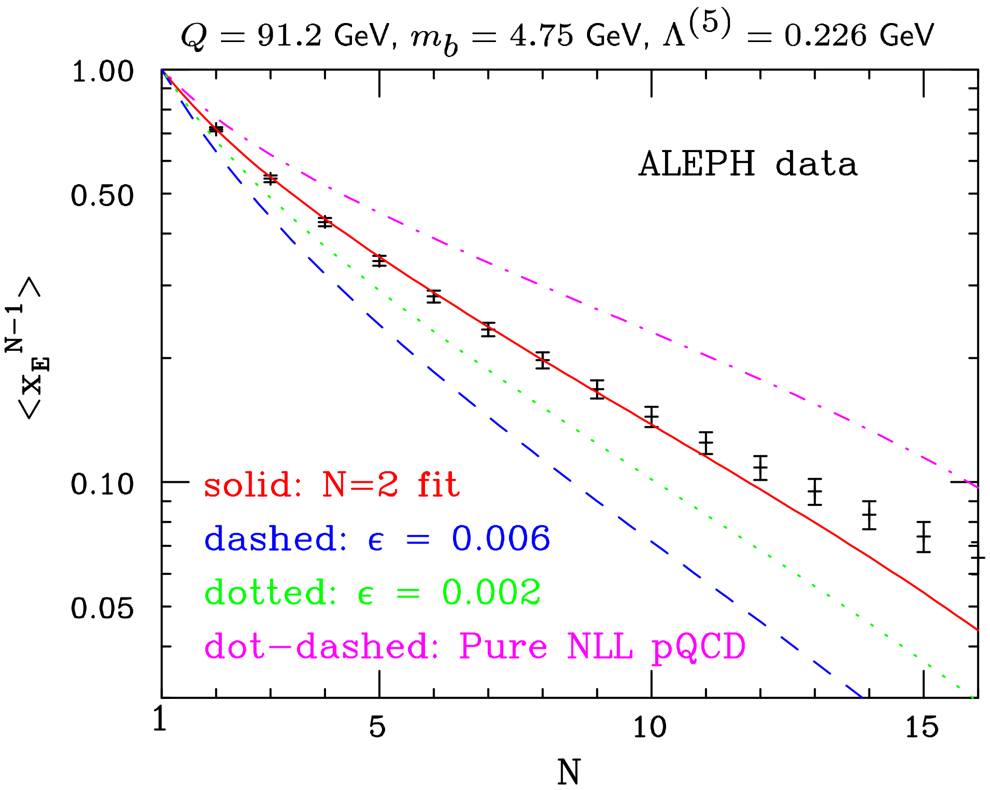
Cacciari & Nason *Phys. Rev. Lett.* **89** (2002) 122003

Only moments of the non-perturbative fragmentation function $N \simeq 5$ are important!

NP information for B–meson fragmentation in moments space

Extracting the non-perturbative correction from $e^+e^-(Q) \longrightarrow B(E) + X$ ALEPH data

$$\frac{1}{\sigma}\frac{d\sigma}{dx}(x,Q^2) = \frac{d\sigma^{\mathrm{PT}}}{dx}(x,Q^2;\mu^2) \otimes D^{\mathrm{NP}}(x;\mu^2), \qquad x \equiv 2E/Q$$

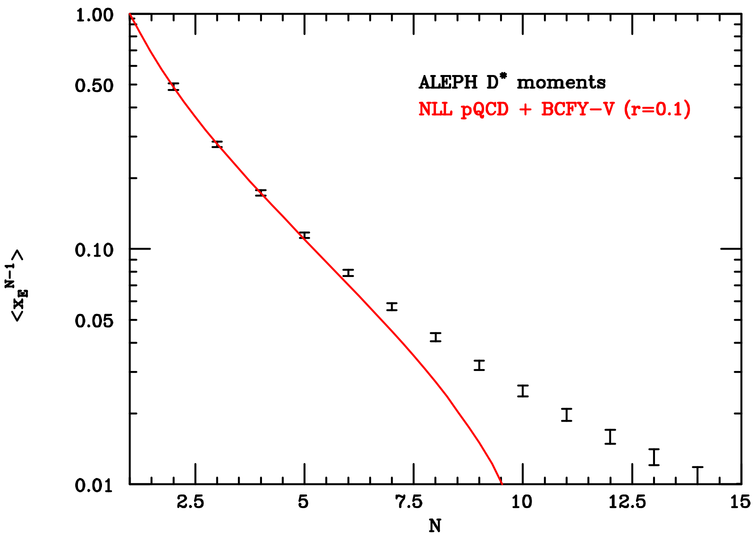


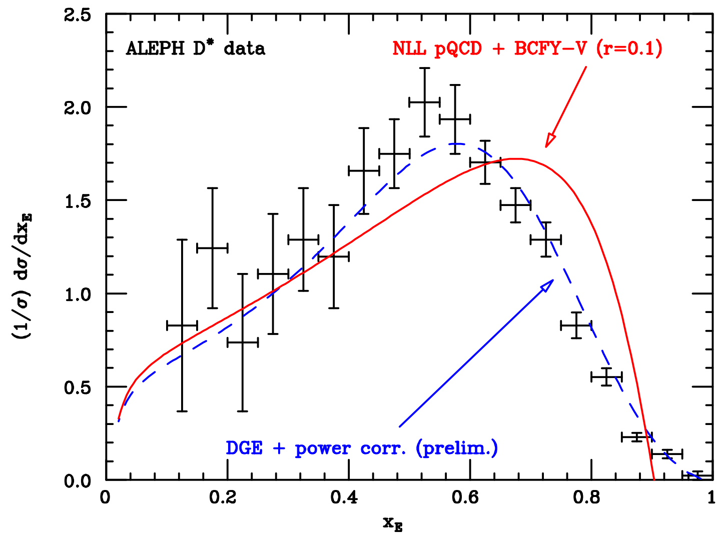
The red solid line is fitted, in moments space, to the $N = 2$ point only $\rightarrow$ this is sufficient to ensure a good description of the first few moments.

A good description is **NOT** obtained by a Peterson model fitted in x space.

Charm fragmentation – new results

NP contribution is fixed so as to describe the moments around N=4,5 in ALEPH data

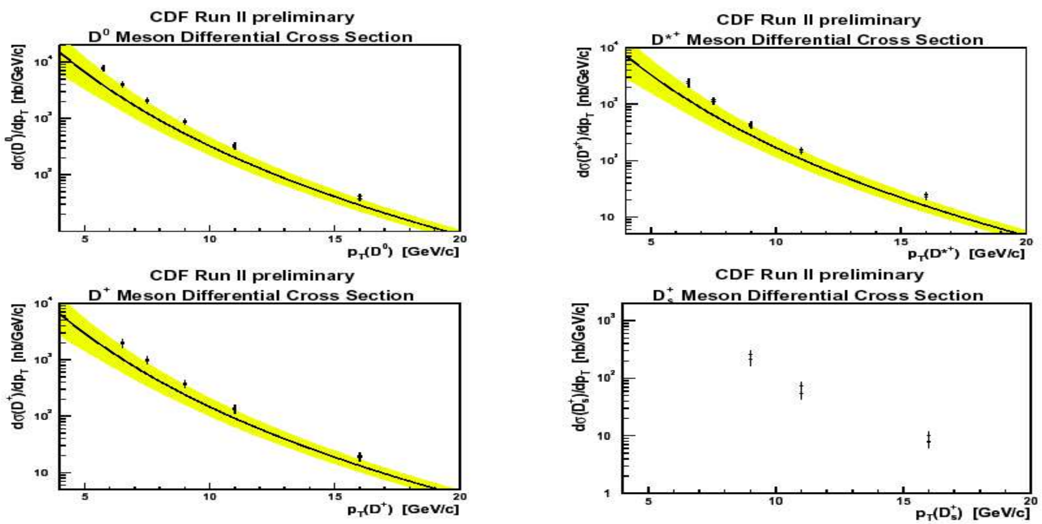




Applied to predict D–production at the Tevatron

Cacciari & Nason, *JHEP* 0309:006 (2003)

Compared with new run II data by CDF (hep-ex/0307080):



Motivation – B production at the Tevatron

- Large discrepancies appear when using today's standard tools
- They are (at least partially) due to naïve application of factorization: the PT/NP separation is delicate
- Moment space analysis is useful
- The main problems appear at large x i.e. high moments

In order to do better we should:

- Carefully separate between PT and NP contributions
- Identify the leading (PT and NP) corrections at large x and resum them

Factorization

- Example: perturbative factorization in $e^+e^-(Q) \longrightarrow B(E)+X$

- Sudakov logs
  in the coefficient function
  and in the fragmentation function

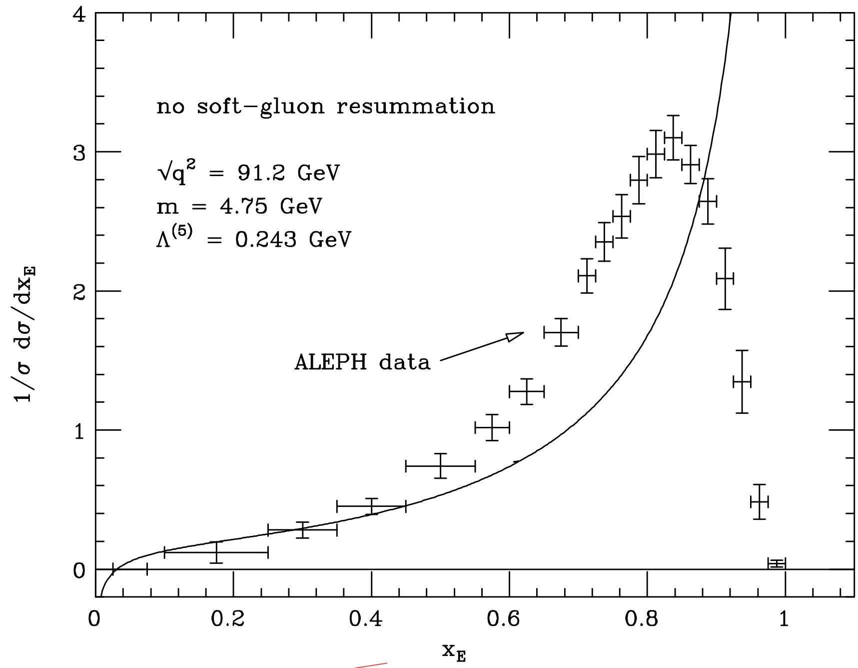
- Renormalons in the Sudakov exponent:
  the origin of the $\mathcal{O}(m(1-z)/\Lambda)$ ambiguity

- The effect of the binding energy
  and Cancellation of ambiguities

- A modified factorization formula

Perturbative calculation of the single-particle differential cross-section $\tilde{\sigma}(N, q^2, m^2)$

NLO result is very far from the data:



~~standard approach: the rest is non-perturbative~~

large corrections need to be identified & resummed

$$\tilde{\sigma}_{\text{PT}}(N, q^2, m^2) = \tilde{C}(N, q^2; \mu_F^2)\, \tilde{E}(N, \mu_F^2, \mu_{0F}^2)\, \tilde{D}_{\text{PT}}(N, m^2; \mu_{0F}^2)$$

(Cacciari & Catani)

$E(N, q^2, m^2)$ is a solution of DGLAP evolution equation,

resumming powers of $\alpha_s \ln m^2/q^2$

$\tilde{C}(N, q^2)$ and $\tilde{D}_{\text{PT}}(N, m^2)$ contain

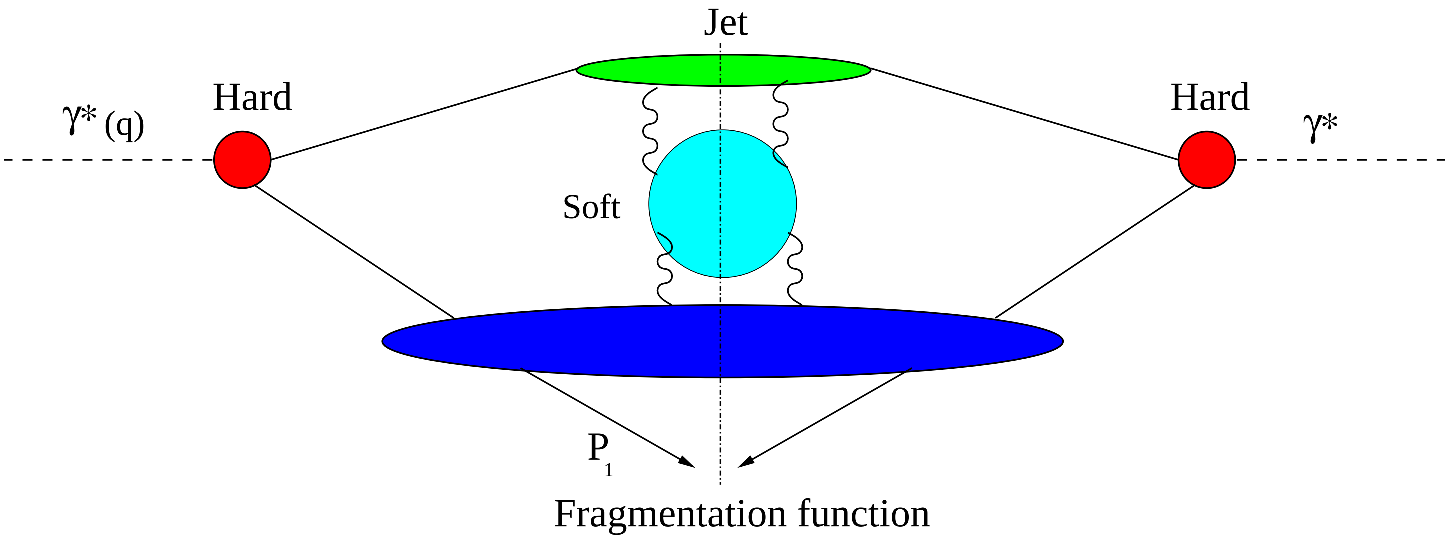
large Sudakov logs: $\alpha_s^n (\ln N)^k$ with $1 \leq k \leq 2n$

infrared renormalons

$e^+e^- \longrightarrow b + X$: Factorization at large x

The relevant scale for $\tilde{D}_{\rm PT}(N, m^2)$ at large N is m/N

The dominant part of $\tilde{C}(N, q^2)$ at large N is the jet function, depending on q^2/N



$$\ln \tilde{\sigma}_{\rm PT}(N, q^2, m^2) = \ln \tilde{J}(N, q^2) + \ln \tilde{E}(N, q^2, m^2) + \ln \tilde{D}_{\rm PT}(N, m^2)$$

multiple soft / collinear radiation $\longrightarrow$ exponentiation

Dressed Gluon Exponentiation

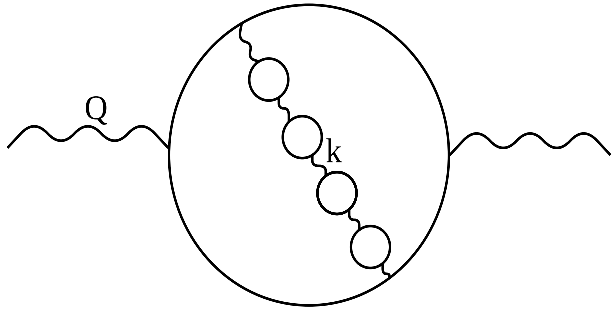
expansion in α_s

large order n →

Single Dressed Gluon

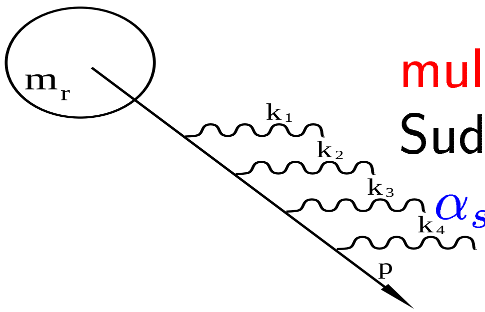
Renormalons $\longrightarrow$ $1/m$

$n! C_F \beta_0^{n-1} \alpha_s^n$



Dressed Gluon Exponentiation (DGE)

$x \longrightarrow 1$ (large N) ↓



multiple emission

Sudakov Double Logs $C_F^n \alpha_s^n (\ln N)^{2n}$

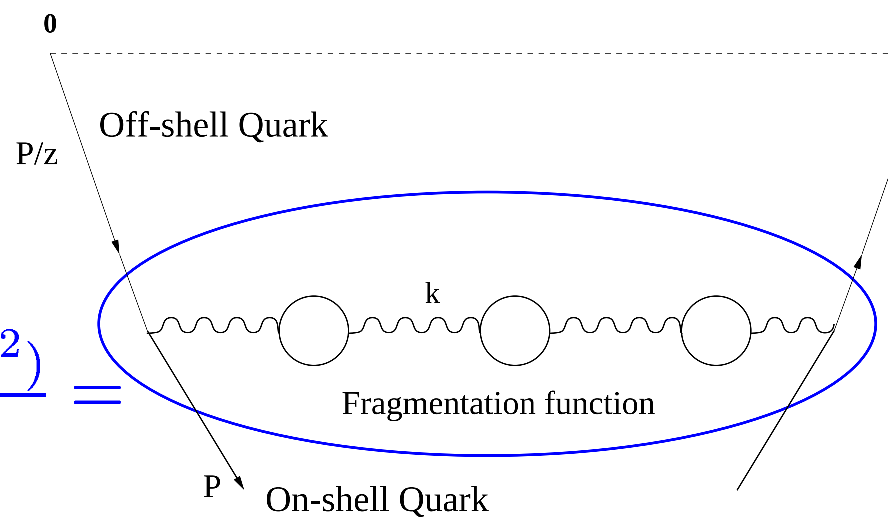
$\alpha_s \ln N \ll 1$

A process-independent calculation of the heavy-quark fragmentation function

All-order calculation in the large-β_0 limit

In light-cone axial gauge $A \cdot y = 0$ the only diagram is:

$$\frac{d\tilde{D}_{\rm PT}(N,m^2)}{d\ln m^2} =$$



$$= -\frac{C_F}{\beta_0}\int_0^\infty du \left(\frac{\Lambda^2}{m^2}\right)^u \int_0^1 dz \, \left(z^{N-1}-1\right)$$

$$\times \left(\frac{z}{(1-z)^2}\right)^u \left[\frac{z}{1-z}(1-u)+\frac{1}{2}(1-z)\,(1+u)\right]$$

$u \longrightarrow \infty$ convergence constraint: $m(1-z) \gtrsim \Lambda$

Infrared renormalons in $\tilde{D}_{\rm PT}(N,m^2)$

The fragmentation function – Resummation and power corrections

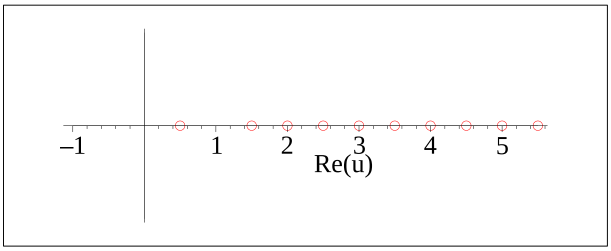
Dressed Gluon Exponentiation

- compute in the large-β_0 limit
- identify $z \longrightarrow 1$ singular terms and exponentiate (N space)
- match with fixed–logarithmic–accuracy and fixed–order results
- identify Borel singularities in the exponent and introduce power corrections

$$\ln \tilde{D}_{\mathrm{PT}}(N, m^2) = \frac{C_F}{\beta_0} \int_0^\infty \frac{du}{u} \left[B_{\mathcal{A}}(u) \ln N - B_{\tilde{D}}^{\mathrm{DGE}}(u, N) \right] \left(\frac{\Lambda^2}{m^2} \right)^u$$

The Borel (u) plane:

u integration needs a prescription (PV)



$$B_{\tilde{D}}^{\mathrm{DGE}}(u, N) = -\,\mathrm{e}^{\frac{5}{3}u}\,(1-u)\,\Gamma(-2u)\,\left(N^{2u}-1\right) + \mathcal{O}\left(\frac{1}{\beta_0}\right)$$

$$\ln \tilde{D}_{\mathrm{NP}}(N\Lambda/m) = -\epsilon_1 \frac{N\Lambda}{m} - \epsilon_3 \left(\frac{N\Lambda}{m}\right)^3 - \epsilon_4 \left(\frac{N\Lambda}{m}\right)^4 + \cdots$$

The effect of the binding energy

Let us write a simplified convolution integral

$$d\sigma(x,Q^2)/dx = \int_x^1 C(z,Q^2)\, D_{\mathrm{NP}}(x/z,m^2)\, dz/z$$

The measured meson has energy $E = \frac{Q}{2}x$ and mass M.
The quark has energy $E_q = \frac{Q}{2}z$ and mass m.
The energy loss depends on the mass ratio m/M.
If $D_{\mathrm{NP}}(x/z,m^2)$ is approximated by a δ function the asymptotic behaviour implies

$$D_{\mathrm{NP}}(x/z,m^2) = \delta\left(\frac{x}{z} - \frac{m}{M}\right)$$

A similar phenomenon occurs in B decay where the difference from 1 account for the energy of the light degrees of freedom. Taking moments

$$\tilde{D}_{\mathrm{NP}}(N,m^2) = \int_0^1 y^{N-1}\delta\left(y - \frac{m}{M}\right) = \left(\frac{m}{M}\right)^{N-1} \simeq \mathrm{e}^{-(N-1)\bar{\Lambda}/M}$$

where $\bar{\Lambda} \equiv M - m$.
This means:

- The physical distribution is *shifted* compared to the PT one.
- In moment space this *shift* appears as *an exponential factor*:

$$\tilde{D}_{\mathrm{NP}}(N,m^2) = \mathrm{e}^{-(N-1)\bar{\Lambda}/M} = \mathrm{e}^{-(N-1)(M-m)/M}.$$

- Different definitions of the quark mass imply different shifts, so they differ by exponential factors in moment space, e.g.

$$D_{\mathrm{NP}}(N,m^2_{\mathrm{pole}}) = D_{\mathrm{NP}}(N,m^2_{\overline{\mathrm{MS}}})\, \mathrm{e}^{(N-1)(m_{\mathrm{pole}}-m_{\overline{\mathrm{MS}}})/M}$$

Cancellation of the leading renormalon ambiguity in the Sudakov exponent

$$\begin{aligned}\widetilde{D}(N;\mu^2) &= \widetilde{D}_{\rm PT}(N, m_{\rm pole}{}^2;\mu^2)\,\times\,\widetilde{D}_{\rm NP}(N, m_{\rm pole}{}^2)\\ &= \widetilde{D}_{\rm PT}(N, m_{\rm pole}{}^2;\mu^2)\,\times\,{\rm e}^{(N-1)\delta m/m}\times\widetilde{D}_{\rm NP}(N, m_{\overline{\rm MS}}{}^2)\\ &= \widetilde{D}_{\rm PT}(N, m_{\overline{\rm MS}}{}^2;\mu^2)\,\times\,\widetilde{D}_{\rm NP}(N, m_{\overline{\rm MS}}{}^2)\end{aligned}$$

$$\delta m/m \equiv (m_{\rm pole} - m_{\overline{\rm MS}})/m_{\overline{\rm MS}}$$

μ^2 represents just logarithmic UV divergence of the matrix element.
$m_{\rm pole}$ has a linear UV divergence, so does $\widetilde{D}_{\rm NP}(N, m_{\rm pole}{}^2)$.
If we use a *short–distance mass*, e.g. $m_{\overline{\rm MS}}$, instead of the pole mass there is an additional contribution from the renormalization of the mass.

$$\ln \tilde{D}_{\rm PT}(N, m^2_{\overline{\rm MS}};\mu^2 = m^2_{\overline{\rm MS}}) = \frac{C_F}{\beta_0}\int_0^\infty du \left(\frac{\Lambda^2}{m^2_{\overline{\rm MS}}}\right)^u \times$$

$$\left\{(N-1)B\left[\frac{\delta m}{m}(u)\right] + \frac{B_{\mathcal{A}}(u)}{u}\ln N - \frac{B^{\rm DGE}_{\tilde{D}}(u,N)}{u}\right\}$$

where $B^{\rm DGE}_{\tilde{D}}(u,N) = -\,{\rm e}^{cu}\,(1-u)\,\Gamma(-2u)\,\left(N^{2u}-1\right)$
$\delta m/m$ has the following large–β_0 Borel transform (Beneke & Braun)

$$B\left[\frac{\delta m}{m}\right](u) = -\frac{3\Gamma(1+u)}{\Gamma(3-u)}\times {\rm e}^{cu}(1-u)\Gamma(-2u) + \frac{1}{4u}\widetilde{G}_{\overline{\rm MS}}(u),$$

$\widetilde{G}_{\overline{\rm MS}}(u)$ has no renormalons.

The singularity at $u=\frac{1}{2}$ cancels between the renormalization of the mass and the large–x limit term. Now the Sudakov exponent is dominated by radiative corrections that are linear in N!!!

A modified factorization formula

To have both PT and NP components well–defined we can incorporate the shift into the convolution integral.
Starting with

$$\frac{d\sigma(E)}{dE} = \int_{2E/Q}^{1} \frac{dz}{z} D(z) \left. \frac{d\sigma^{\rm PT}}{dE_q} \right|_{E_q=E/z}$$

using the asymptotic behaviour

$$D(z) = \frac{M}{\bar{\Lambda}} f\left(\frac{(1-z)M}{\bar{\Lambda}}\right)$$

and changing variables to $y \equiv (1-z)M/\bar{\Lambda}$

$$\bar{D}(y) \equiv \frac{1}{1-y\bar{\Lambda}/M} f(y) = \frac{\bar{\Lambda}/M}{1-y\bar{\Lambda}/M} D(1-y\bar{\Lambda}/M)$$

where $\bar{D}(y)$ has support for $0 < y < \infty$.
We get a modified factorization formula,

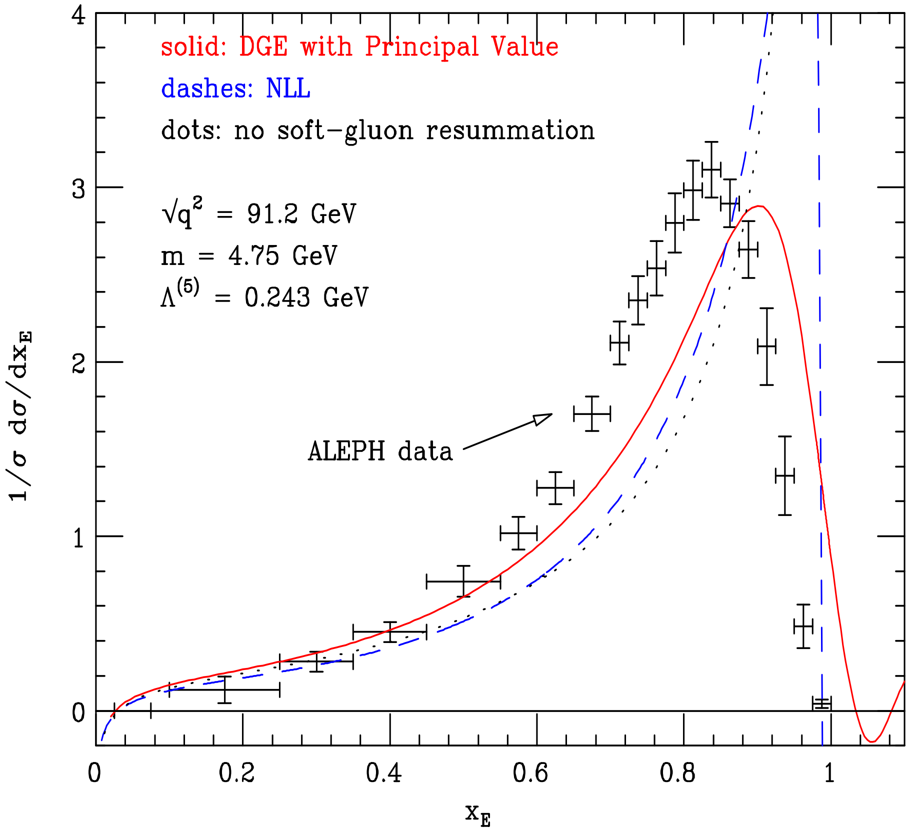
$$\frac{d\sigma(E)}{dE} = \int_{0}^{(1-2E/Q)M/\bar{\Lambda}} dy \bar{D}(y) \left. \frac{d\sigma^{\rm PT}}{dE_q} \right|_{E_q = \frac{E}{1-y\bar{\Lambda}/M}}$$

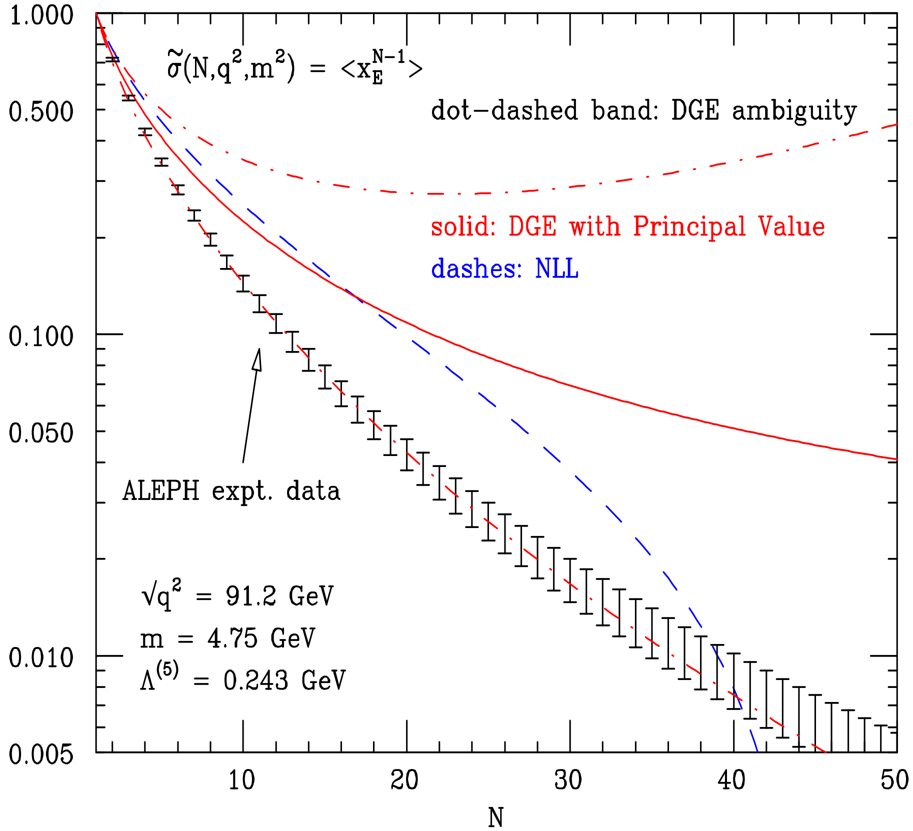
where *each component is well–defined*: no $\mathcal{O}(N\Lambda/M)$ ambiguity.
$\bar{D}(y)$ is a shape function: it only *smears* the perturbative distribution without shifting it.

Results of Dressed Gluon Exponentiation (DGE)

- Significant effect of Sudakov logs and $\mathcal{O}(m(1-z)/\Lambda)$ terms
- Absence of Landau singularity in PV regularization — a possibility to describe the data to high moments
- Interpretation of the measured shift as $\bar{\Lambda} = M - m_{\text{pole}}$
- Universality

The Dressed Gluon Exponentiation result



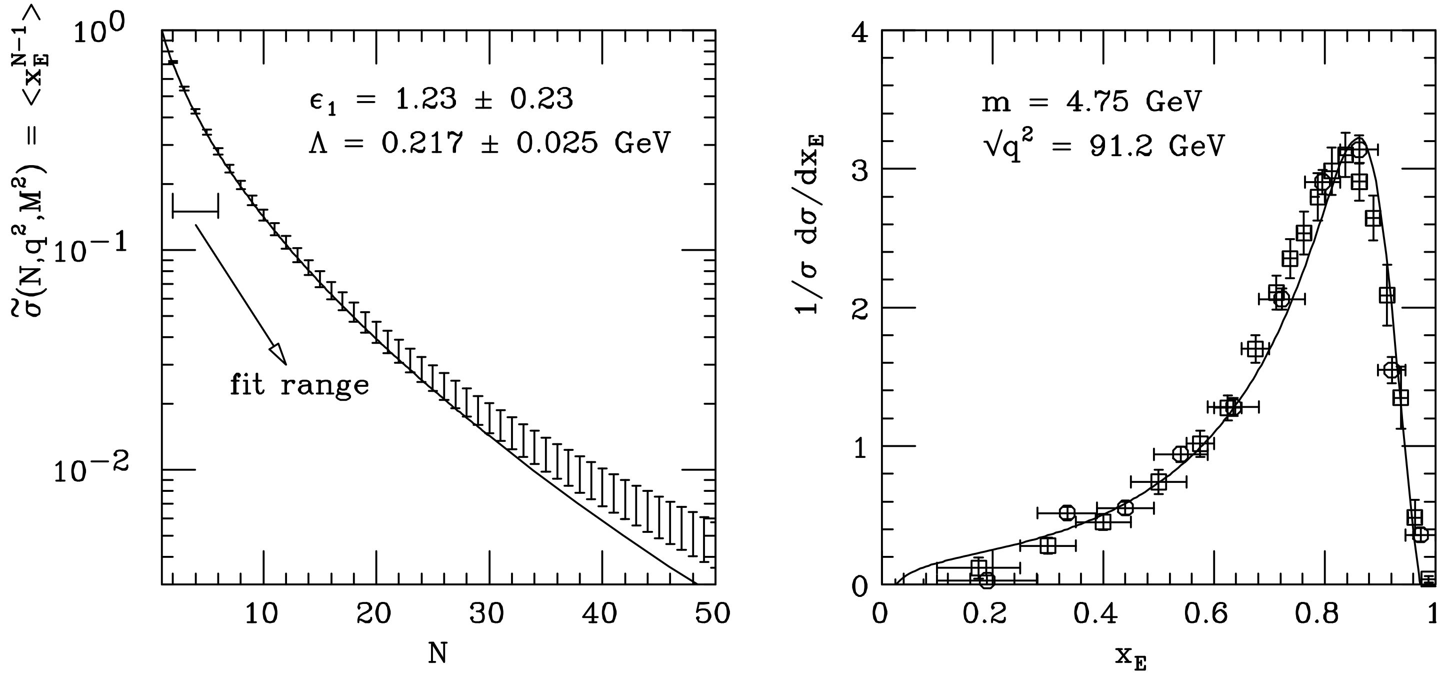


$$\tilde{D}(N,m^2) = \tilde{D}_{\mathrm{PT}}(N,m^2)\,\tilde{D}_{\mathrm{NP}}(N,m^2) = \tilde{D}_{\mathrm{PT}}(N,m^2)\,\mathcal{F}(N\Lambda/m)$$

$\tilde{D}_{\mathrm{PT}}(N,m^2)$ is defined by Principal Value. The result has no Landau singularities!

$$\tilde{D}_{\mathrm{NP}}(N\Lambda/m) = \exp\left\{-\epsilon_1\frac{N\Lambda}{m} - \epsilon_3\left(\frac{N\Lambda}{m}\right)^3 - \epsilon_4\left(\frac{N\Lambda}{m}\right)^4 + \cdots\right\} \qquad \epsilon_1\Lambda = \bar{\Lambda}_{\mathrm{pole(PV)}} \equiv M - m_{\mathrm{pole(PV)}}$$

A DGE shift–based fit



- A shift–based fit involves one non-perturbative parameter: the binding energy

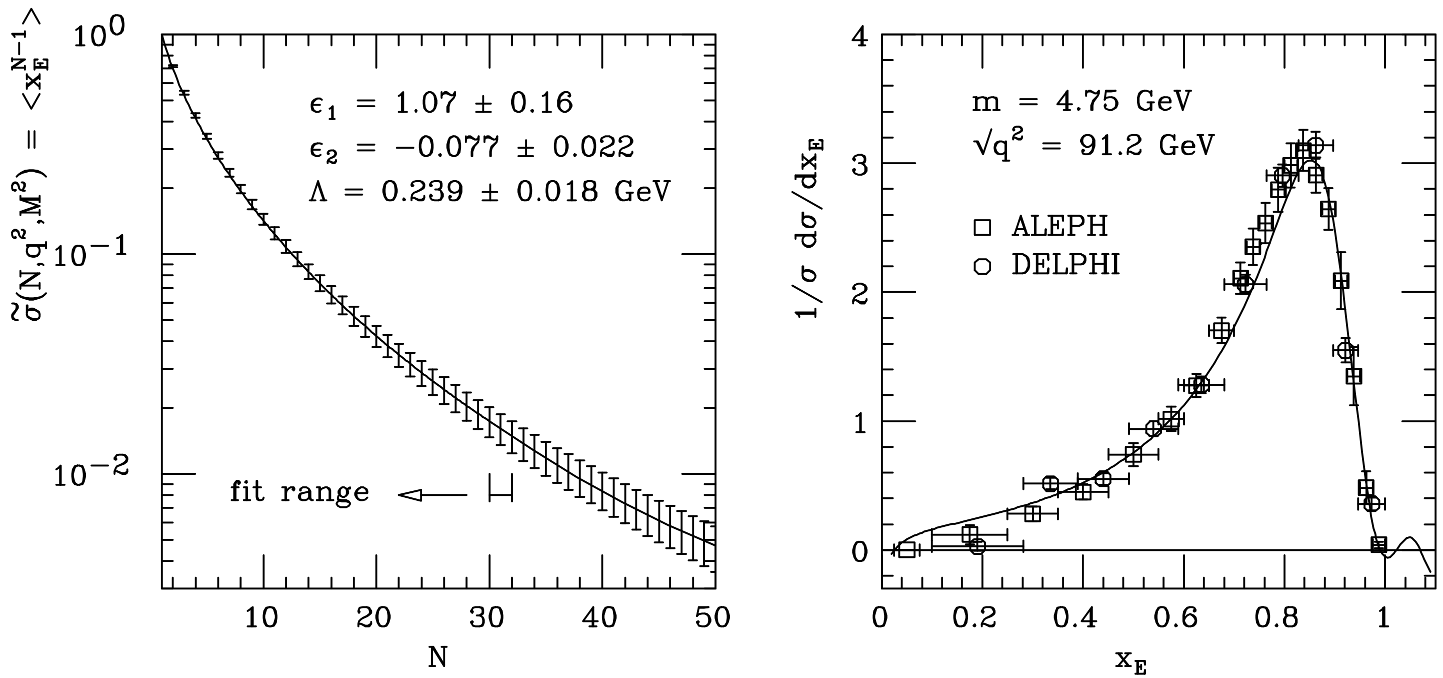
$$\epsilon_1 \Lambda = \bar{\Lambda}_{\text{pole(PV)}} \equiv M_B - m_{\text{pole(PV)}}$$

- The measured value

$$\bar{\Lambda}_{\text{pole(PV)}} \simeq 250 \pm 50 \text{ MeV}$$

- $\bar{\Lambda}_{\text{pole(PV)}}$ is formally independent of m in the large–mass limit (with $\mathcal{O}(\Lambda/m)$ accuracy)

A DGE based fit for high moments with some smearing



- fit at very high moments $N = 29, 30, 31$ is still consistent with fits at low N.
- The measured smearing (ϵ_2) is small.

Conclusions

- Asymptotic scaling: $\tilde{D}(N, m^2) = \mathcal{F}(N\Lambda/m) + \mathcal{O}(1/N)$
  The heavy–quark fragmentation process is dominated by the scale m/N.
- Comparison with experimental data shows
  - large discrepancies appear when using today's standard tools
  - they are (at least partially) due to naïve application of factorization: the PT/NP separation is delicate
  - moment space analysis is useful
  - the main problems appear at large x i.e. high moments
- Our approach to heavy quark fragmentation
  - To control $\mathcal{O}(N\Lambda/m)$ effects two concepts should be combined (DGE): renormalons and Sudakov resummation
  - Resummation + power corrections replace models
- Theoretically the origin and eventual cancellation of the leading renormalon $\mathcal{O}(N\Lambda/m)$ is now understood
  - A shift–based fit involves one non-perturbative parameter: the binding energy

$$\epsilon_1 \Lambda = \bar{\Lambda}_{\text{pole(PV)}} \equiv M_B - m_{\text{pole(PV)}}$$

  - The measured value

$$\bar{\Lambda}_{\text{pole(PV)}} \simeq 250 \pm 50 \text{ MeV}$$

  - $\bar{\Lambda}_{\text{pole(PV)}}$ is formally independent of m in the large–mass limit (with $\mathcal{O}(\Lambda/m)$ accuracy)
- A modified factorization formula has been suggested, where the binding energy is treated separately from sub-leading NP effects.
- Bottom and Charm production spectra from LEP are well described
- Process independence is expected