

CASCADE/CCFM predictions and heavy quark data

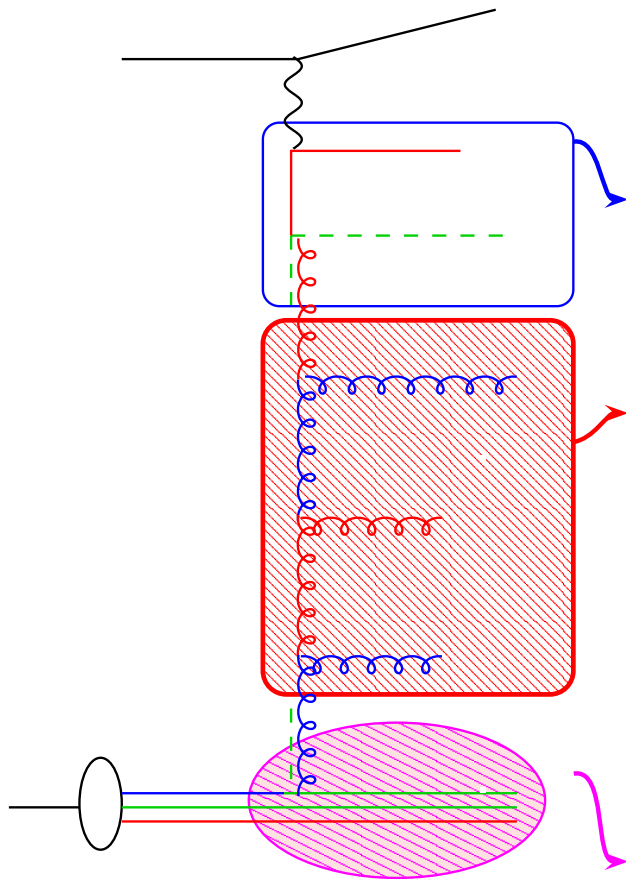
H. Jung, University of Lund

Heavy Quark Physics at the upgraded HERA Collider, Weizmann Institute, 19-22 Oct 2003

- basics, factorisation and all that ...
- CCFM equation, solution, new hadron level MC CASCADE
one-loop (DGLAP) - all loops (CCFM)
- new fits to F_2 - new unintegrated gluon densities
- predict heavy quark production
at HERA, Tevatron and at LEP ($\gamma\gamma \rightarrow b\bar{b}$) ???
- some ideas to measure unintegrated partons
- conclusion

Basic idea - Collinear factorization

DGLAP



BGF matrix element
on mass shell

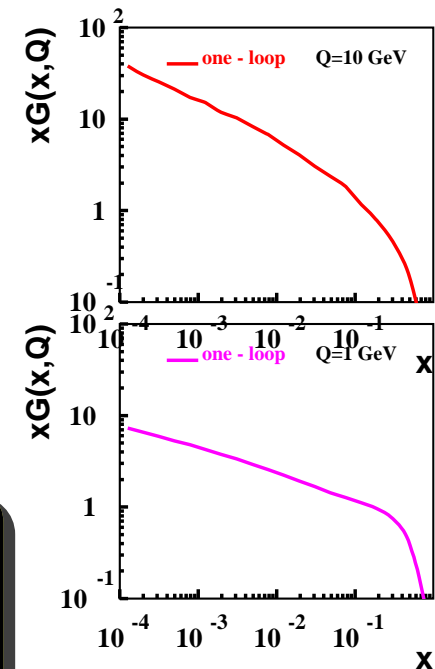
evolution of parton cascade
 q - ordered
with DGLAP splitting fct.

$$\tilde{P}(z, q, k_t) = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \right)$$

initial distribution: steep

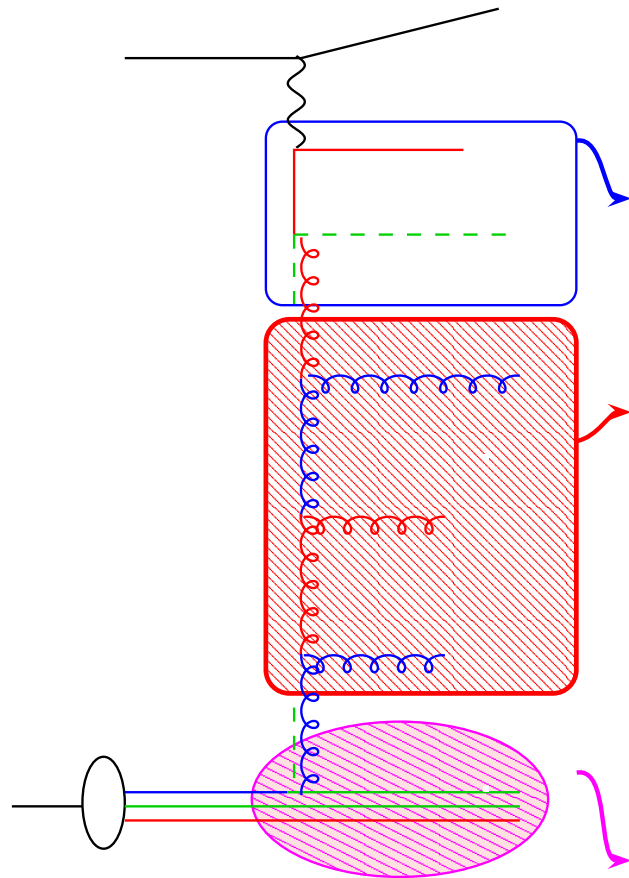
$$\sigma(ep \rightarrow e' q \bar{q}) = \int \frac{dy}{y} d^2 Q \frac{dx_g}{x_g} \hat{\sigma}(\hat{s}, 0, Q) \int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with $\int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) = x_g G(x_g, Q^2)$



Basic idea - Collinear factorization

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BGF matrix element
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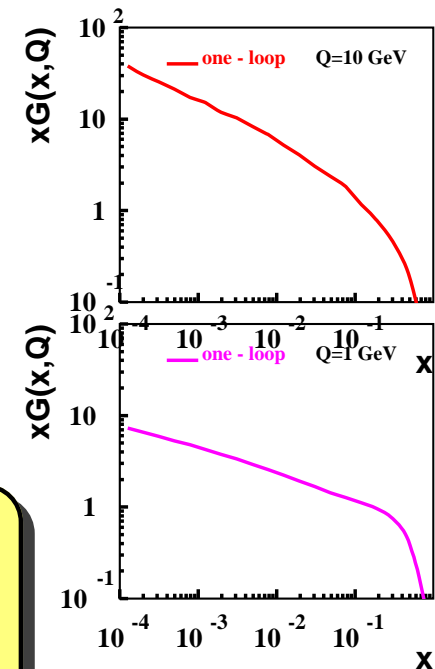
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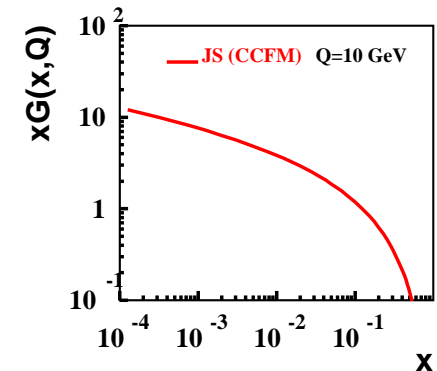
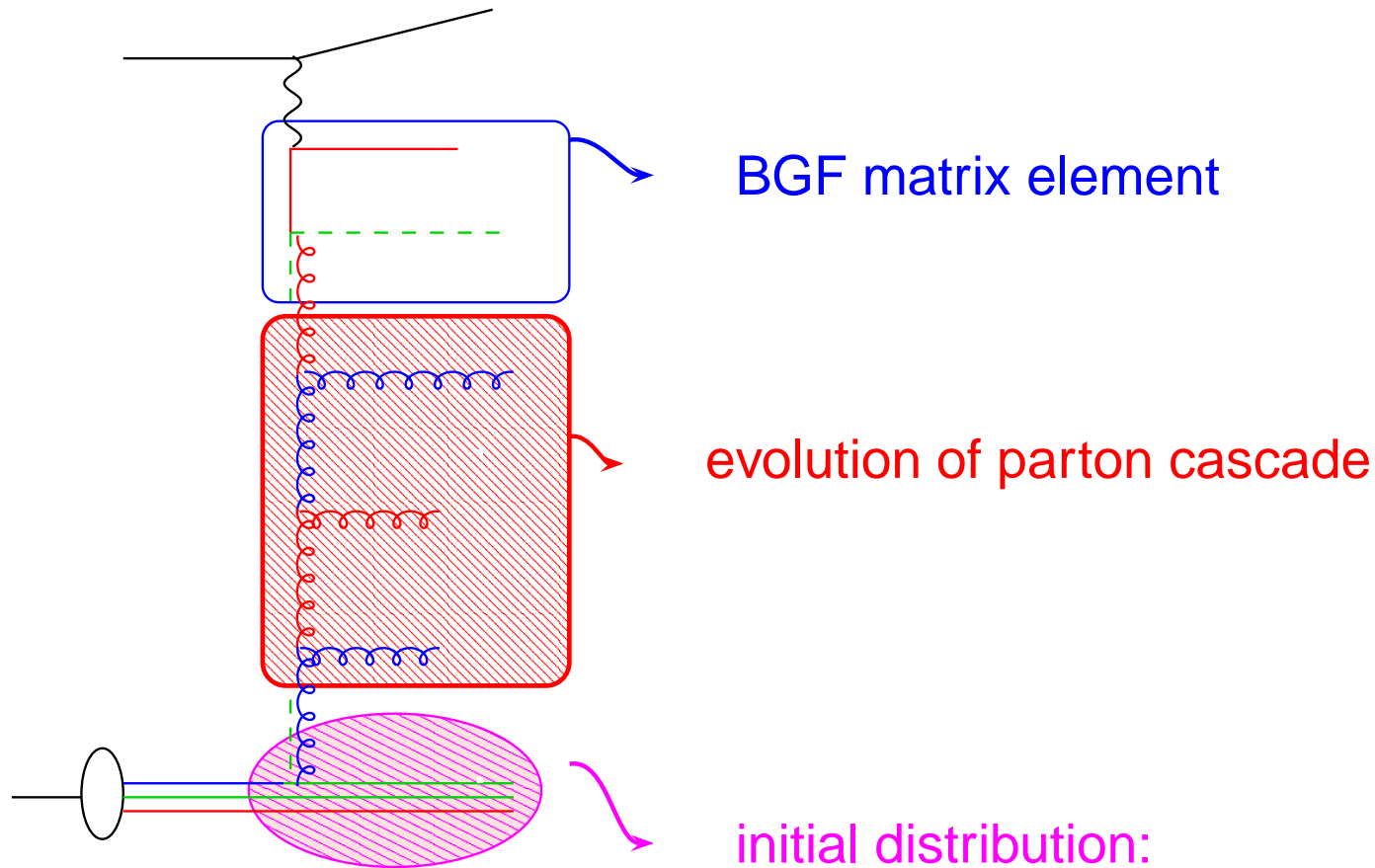
$$\sigma(ep \rightarrow e' q \bar{q}) = \int \int d^2 k_t \frac{dy}{y} d^2 Q \frac{dx_g}{x_g} \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with $\int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \stackrel{?}{=} x_g G(x_g, Q^2)$

J.C. Collins, X. Zu JHEP 06 (2002) 018
on shell matrix element
is only assumption without
proof
and is unphysical !!!
and not necessary for proof
of factorization !!!



Basic idea - k_t factorization

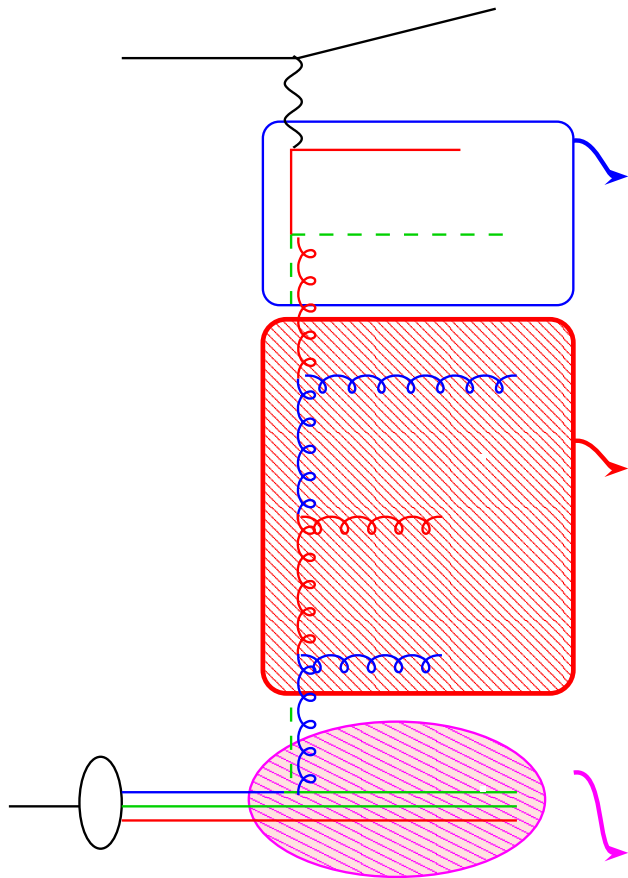


$$\sigma(ep \rightarrow e'q\bar{q}) = \int \frac{dy}{y} d^2 Q \frac{dx_g}{x_g} \int d^2 k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with $\int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$

Basic idea - k_t factorization

CCFM



BGF matrix element
off mass shell

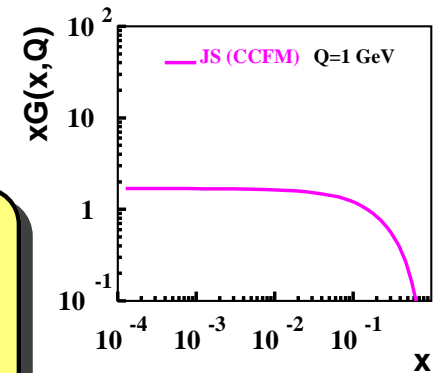
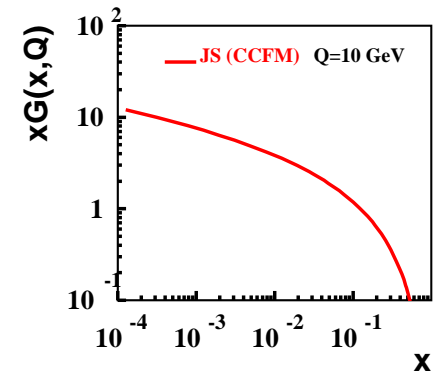
evolution of parton cascade
with CCFM splitting fct.

$$\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \Delta_{ns} + \dots \right)$$

initial distribution: flat ?

CCFM !!!

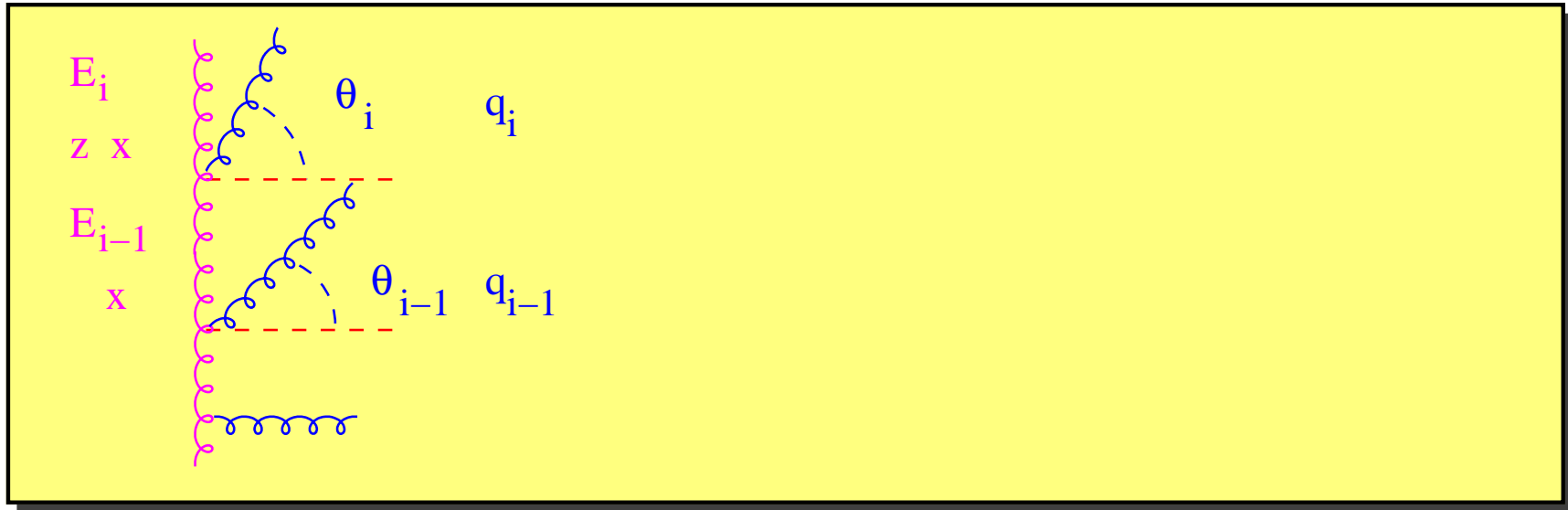
- angular ordering
(instead of q_t ordering)
- Δ_{ns} (non - Sudakov)



$$\sigma(ep \rightarrow e'q\bar{q}) = \int \frac{dy}{y} d^2 Q \frac{dx_g}{x_g} \int d^2 k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with $\int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:



- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL



WOW

for small z no restriction in q :  random walk in q

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:

E_i
 z x
 θ_i q_i
 E_{i-1}
 x
 θ_{i-1} q_{i-1}

$$p_{ti} = |q_i^0| \sin \Theta_i, z = \frac{E_i}{E_{i-1}}$$

$$E_{i-1} = E_i + q_i^0 = z E_{i-1} + q_i^0, \Rightarrow q_i^0 = (1 - z) E_{i-1}$$

$$p_{ti} = q_i^0 \sin \Theta_i \simeq (1 - z) E_{i-1} \Theta_i$$

$$\frac{p_{ti}}{1-z} \simeq E_{i-1} \Theta_i$$

with: $q_i = \frac{p_{ti}}{1-z_i} \Rightarrow \Theta_i = \frac{q_i}{E_{i-1}}$ and $\Theta_{i+1} = \frac{q_{i+1}}{E_i}$

- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL



WOW

for small z no restriction in q : random walk in q

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:

in lab. frame

$$\Theta_{i+1} > \Theta_i$$

$$q_{i+1} > z_i q_i$$

with $q = \frac{p_t}{1-z}$

- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL



WOW

for small z no restriction in q : random walk in q

CCFM equation: small and large x

$$\mathcal{A}(x, k_t, \bar{q}) = \mathcal{A}_0(x, k_t) \Delta_s(\bar{q}, Q_0) + \int \frac{dz}{z} \int \frac{d^2q}{\pi q^2} \Theta(\bar{q} - zq) \cdot \Delta_s(\bar{q}, zq) \tilde{P}(z, q, k_t) \mathcal{A}\left(\frac{x}{z}, k'_t, q\right)$$

CCFM Splitting fct: $\tilde{P}(z, q, k_t) = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\text{ns}}(z, q, k_t)$

Sudakov $\Delta_s(a, b)$: **probability for no radiation in $[a, b]$**

angular ordering: $\bar{q} > z_n q_n, q_n > z_{n-1} q_{n-1}, \dots, q_1 > Q_0$

small x

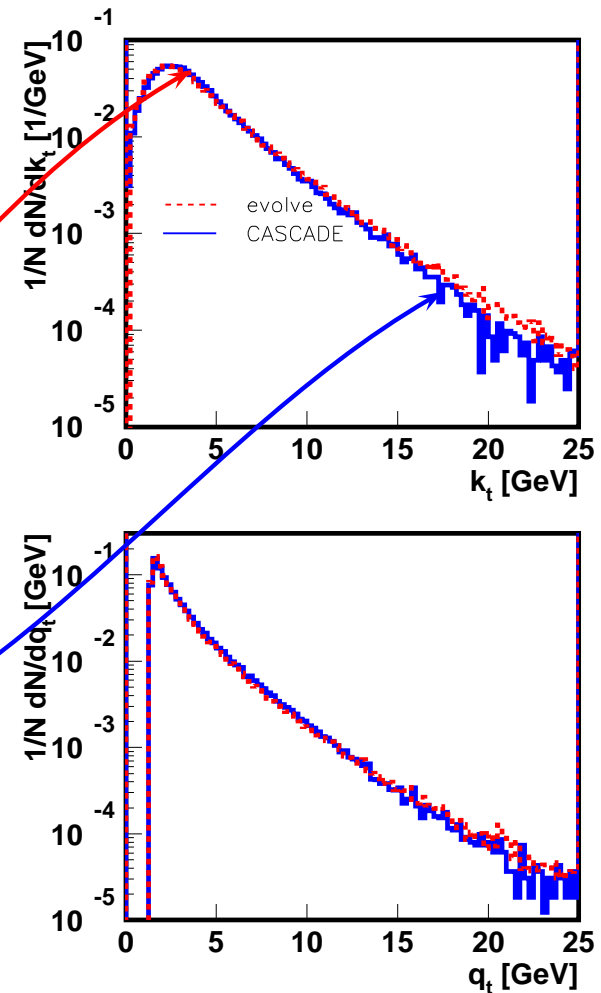
- ➔ **BFKL limit ($z \rightarrow 0$)**
- ➔ **angular ordering**
- ➔ **no restriction on q_i**

large x

- ➔ **DGLAP limit ($z \gg 0$)**
- ➔ **DGLAP splitting fct \tilde{P} with $\Delta_{\text{ns}} = 1$**
- ➔ **angular ordering $\rightarrow q_i$ ordering**

Advantage of CCFM: parton emissions

- DGLAP or BFKL
- ☞ only inclusive predictions
- ☞ no info on emitted partons !!!
- CCFM treats explicitly:
 - partons emitted during cascade
 - color coherence
 - energy momentum conservation
- best to implement in MC generator
- ☞ compare **evolution** and MC
- CASCADE MC generator



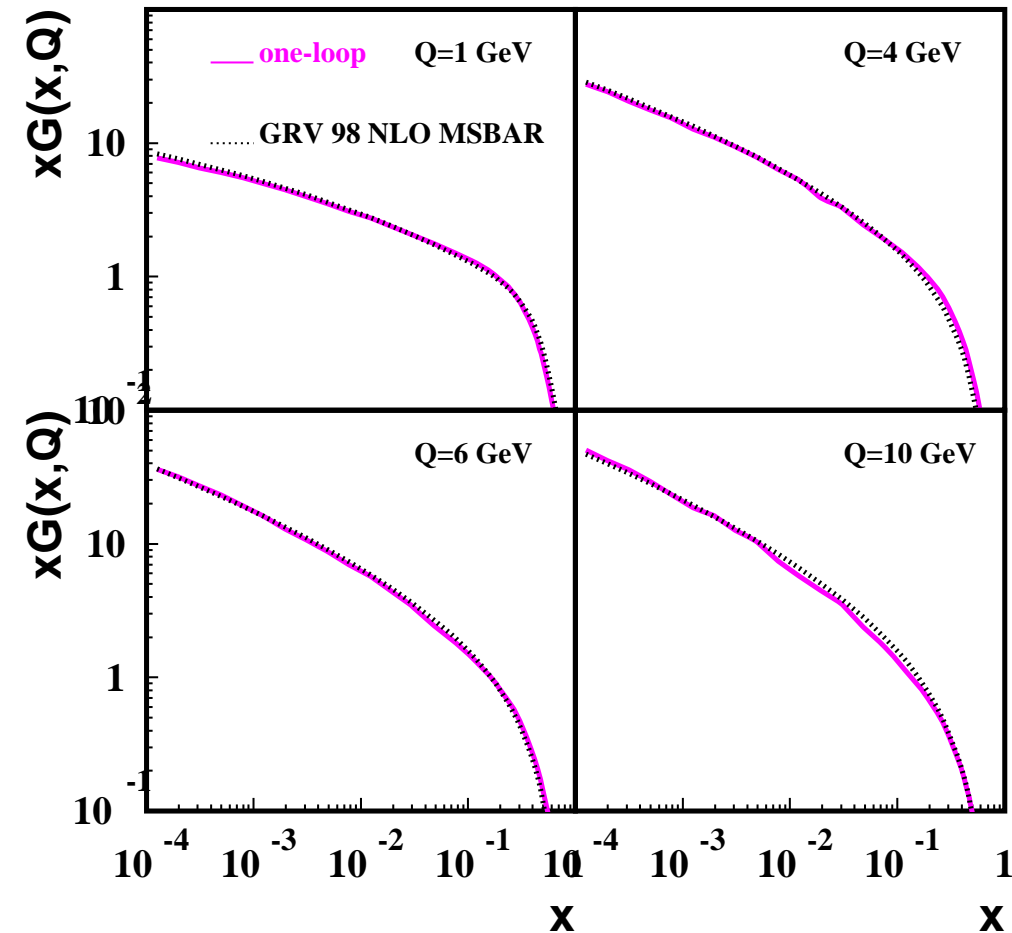
evolution - MC parton shower comparison
never shown for DGLAP type MC's!!!

The Monte Carlo Generator CASCADE

- CCFM backward evolution implemented in MC generator **CASCADE** (<http://www.quark.lu.se/hannes/cascade>)
- initial state CCFM cascade with strict angular ordering
- off-shell hard scattering processes:
 - ☞ $\gamma g^* \rightarrow q\bar{q}, \gamma^* g^* \rightarrow Q\bar{Q}, \gamma g^* \rightarrow J/\psi g, \gamma\gamma \rightarrow Q\bar{Q}$
 - ☞ $g^* g^* \rightarrow q\bar{q}, g^* g^* \rightarrow Q\bar{Q}, g^* g^* \rightarrow h$
- *P*-remnant treatment like in PYTHIA (*q*-di-*q*, primordial k_t)
- final state parton showers added to quarks hadronization via JETSET/PYTHIA

CASCADE is MC implementation of CCFM
for $ep, ee, \gamma\gamma$ and also for $p\bar{p}$

DGLAP unintegrated gluon density - integrated -



one-loop gluon integrated over k_t

$$\int_0^{\bar{q}} dk_t^2 x \mathcal{A}(x, k_t, \bar{q}) = xG(x, \bar{q})$$
➤ compare to evolved DGLAP gluon

one-loop gluon:

- at starting scale use GRV
- full treatment of kinematics
- good agreement with full splitting fct
but also with $\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \right)$
- evolution machinery works perfectly

Non-Sudakov and all - loop resummation

Splitting Fct: $\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\text{ns}}(z, q, k_t)$

Non - Sudakov form factor \blacktriangleright **all loop resummation:**

$$\Delta_{\text{ns}} = \exp \left[-\bar{\alpha}_s(k_t^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_t - q) \Theta(q - z' q_t) \right]$$

$$\Delta_{\text{ns}} = 1 + \left(-\bar{\alpha}_s(k_t^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2} \right)^1 + \frac{1}{2!} \left(-\bar{\alpha}_s(k_t^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2} \right)^2 \dots$$

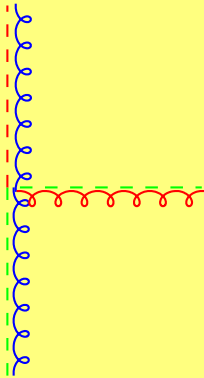
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$$\bar{\alpha}_s(k_t) \frac{1}{z} \left[1 \right]$$

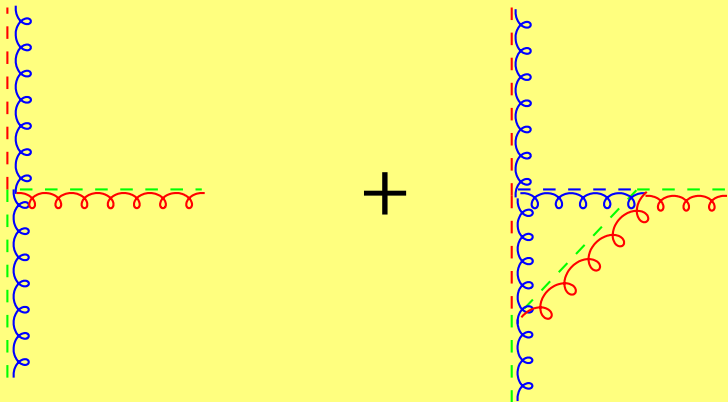
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$$\bar{\alpha}_s(k_t) \frac{1}{z} \left[1 + \bar{\alpha}_s \log \left(\frac{z}{z_0} \right) \log \left(\frac{k_t^2}{z_0 z q^2} \right) \right]$$

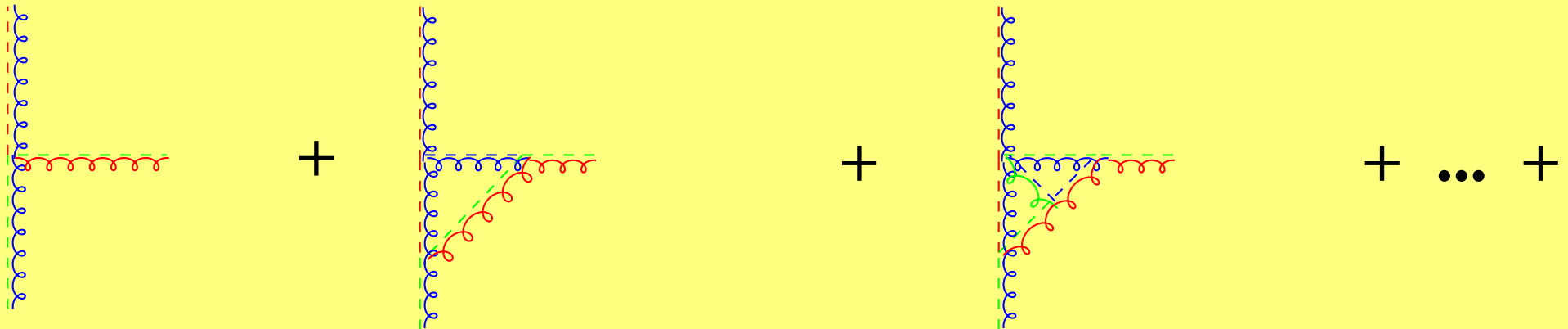
Non-Sudakov and all - loop resummation

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$$\bar{\alpha}_s(k_t) \frac{1}{z} \left[1 + \bar{\alpha}_s \log \left(\frac{z}{z_0} \right) \log \left(\frac{k_t^2}{z_0 z q^2} \right) + \frac{1}{2!} \left(\bar{\alpha}_s \log \left(\frac{z}{z_0} \right) \log \left(\frac{k_t^2}{z_0 z q^2} \right) \right)^2 \dots \right]$$

Structure Function $F_2(x, Q^2)$

together with G.P. Salam, EPJC 19, 351 (2001)

With $\sigma = \int dk_t^2 dx_g \mathcal{A}(x_g, k_t^2, \bar{q}) \sigma(\gamma^* g^* \rightarrow q\bar{q})$ fit $F_2(x, Q^2)$

(data from H1 Coll, NPB 470 (1996) 3.)

Parameters in fit

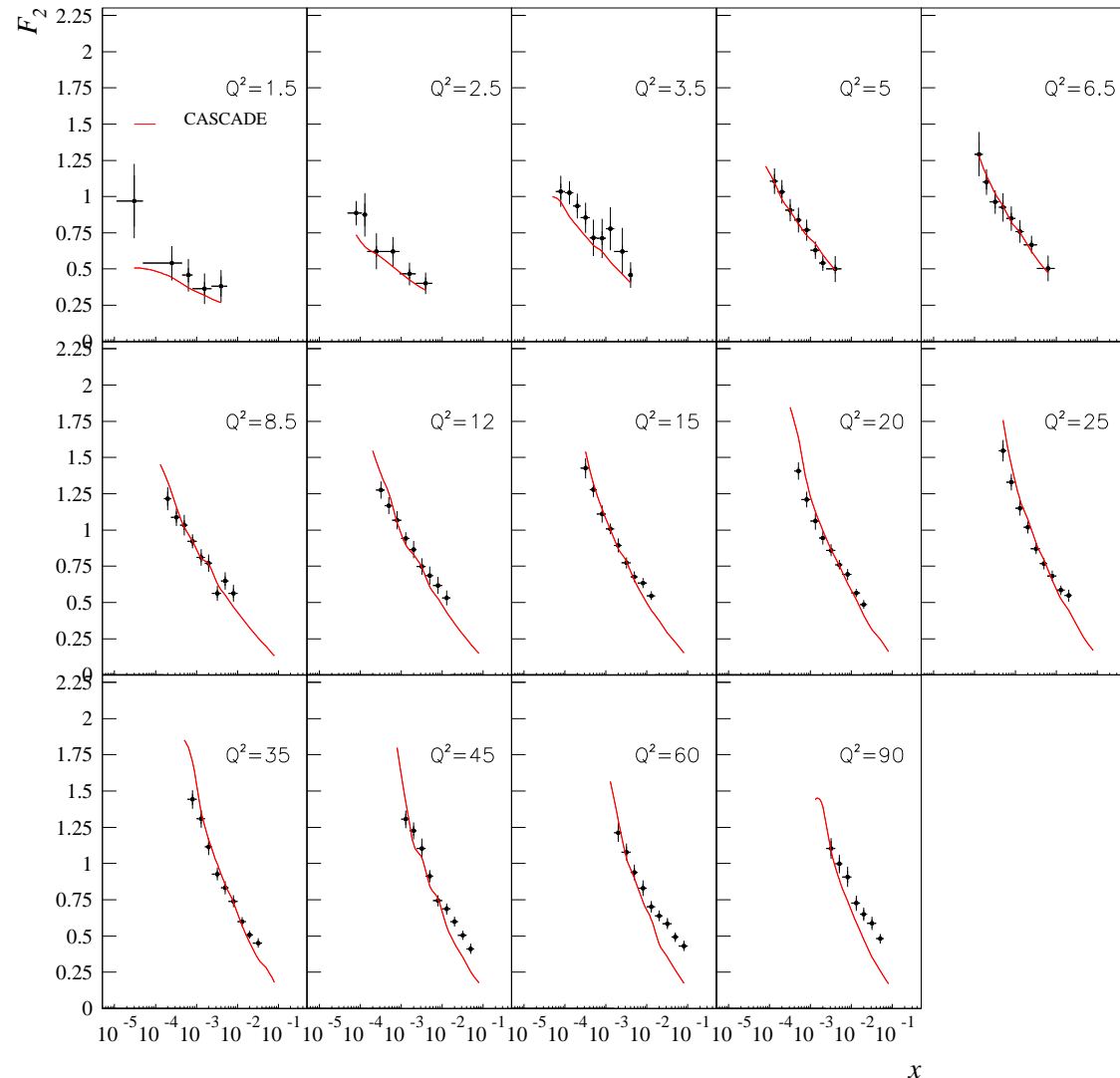
(fitted for $Q^2 > 5 \text{ GeV}^2$, $x < 10^{-2}$)

- collinear cut-off
 $Q_0 = 1.4 \text{ GeV}$
- initial gluon $x\mathcal{A}_0(x, k_{t0}^2)$
- freezing of $\alpha_s(k_t)$ for
 $k_t \rightarrow 0$
 k_t not constrained ...
- light quark masses:
 $m_q = 0.250 \text{ GeV}$,
 $m_c = 1.5 \text{ GeV}$

unintegrated gluon density

$$x\mathcal{A}(x, k_t^2, \bar{q})$$

obtained from fit to F_2



New fit: small k_t - region

- use H1 + ZEUS F_2 data (from 94 and 96-97)
- fit for $x < 0.005$ $Q^2 > 4.5 \text{ GeV}^2$
- fit Q_0 and normalization in initial pdf $x\mathcal{A}_0 = N(1-x)^4$

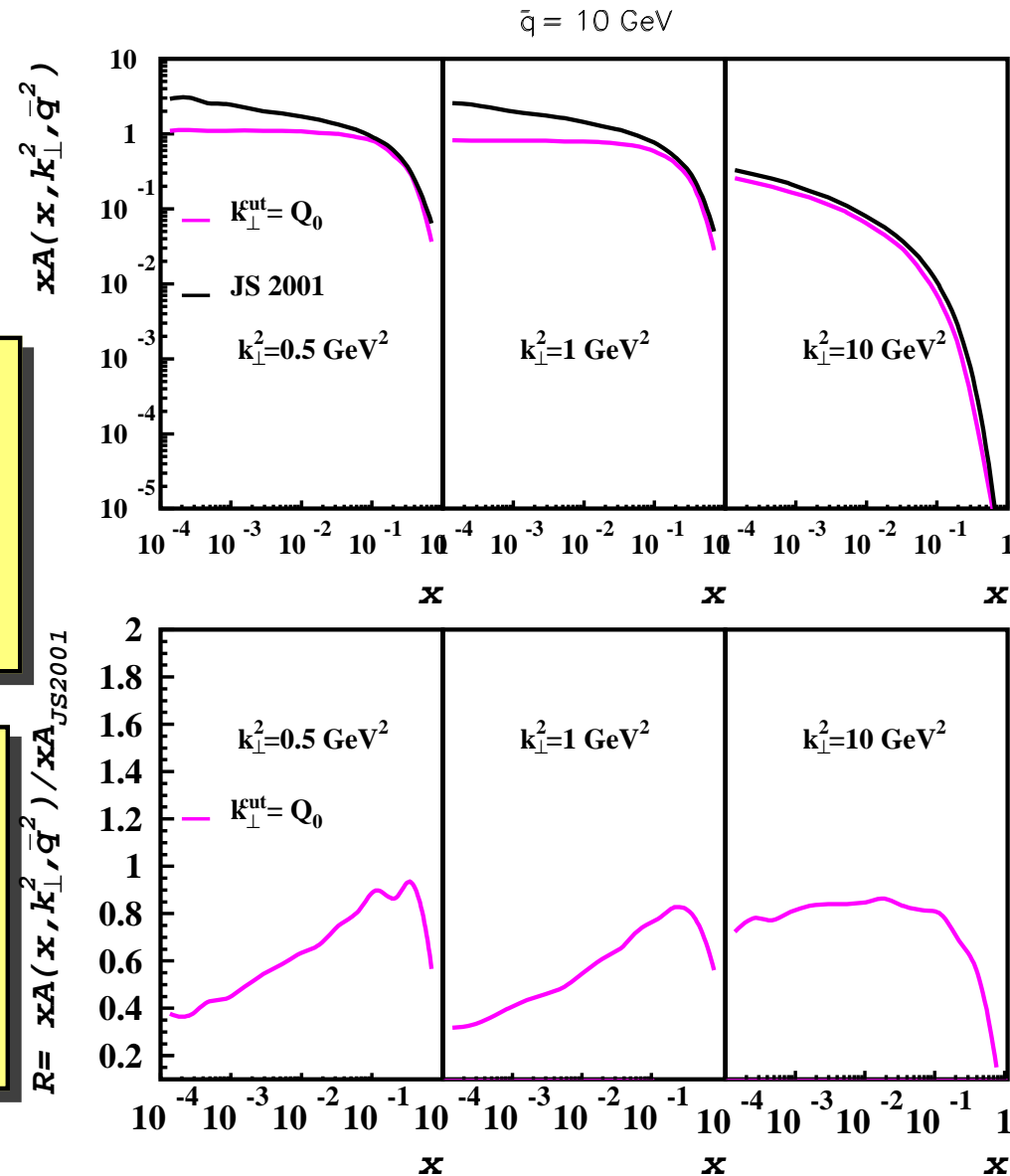
Treatment of soft region

no k_t ordering \rightarrow diffusion into soft

- what about α_s for $k_t < k_t^{cut}$ in
- \rightarrow saturation of x-section for $k_t < k_t^{cut}$

What is soft?

- JS2001 $k_t^{cut} > 0.25 \text{ GeV}$
- now $k_t^{cut} > Q_0$
- similar to saturation scale $Q_s \sim 1 \text{ GeV}$



New fit: full splitting function

- improve splitting function

$$P_{gg} \sim \bar{\alpha}_s \left(\frac{1}{z} \Delta_{ns} + \frac{1}{1-z} \right)$$

- to include non-singular terms

$$\frac{1}{z} \Delta_{ns} - 2 + z(1-z) + \frac{1}{1-z}$$

???

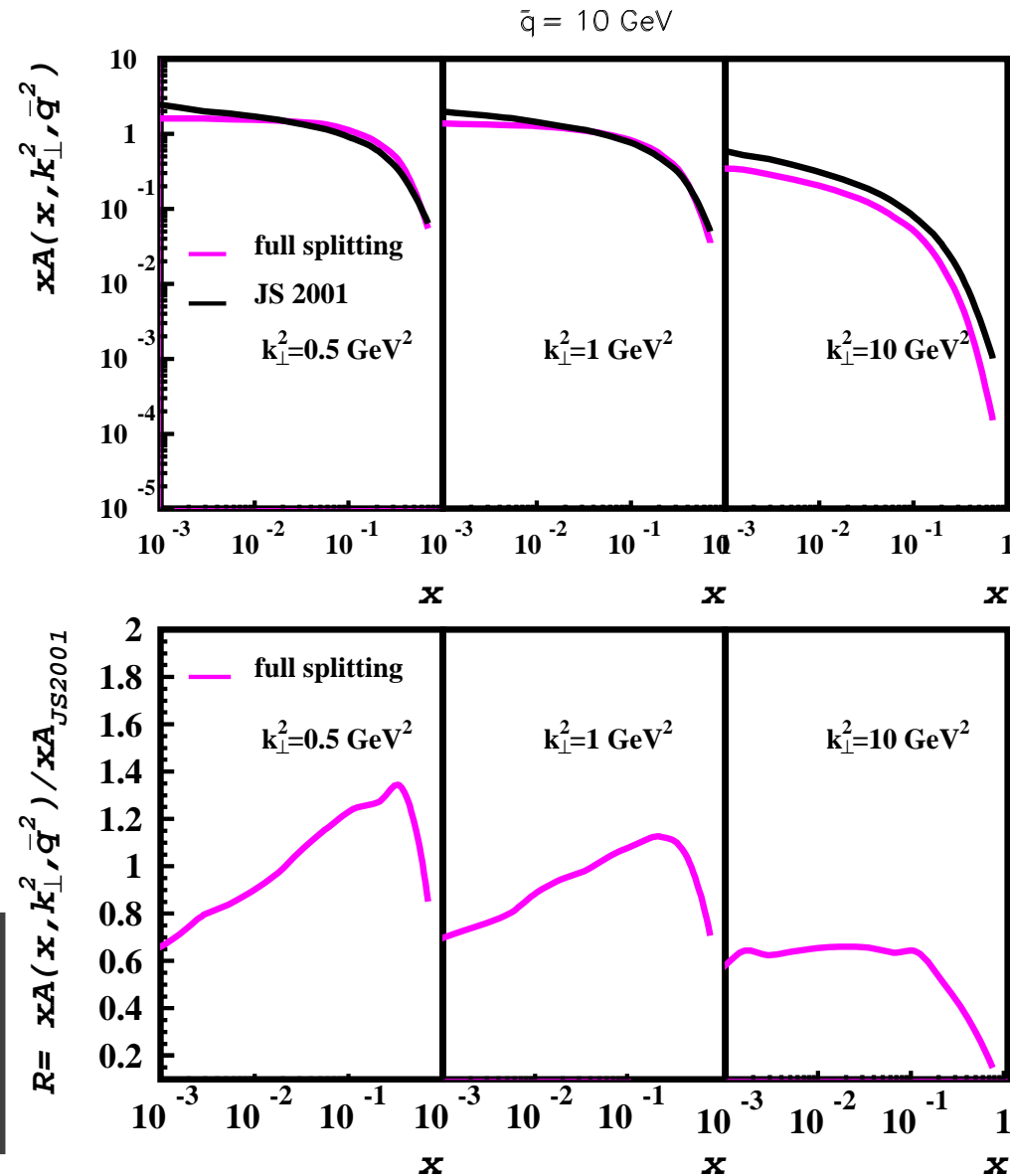
- new

$$P = \bar{\alpha}_s \left(\frac{(1-z)}{z} + \frac{z(1-z)}{2} \right) \Delta_{ns} + \bar{\alpha}_s \left(\frac{z}{1-z} + \frac{z(1-z)}{2} \right)$$

- need also new Sudakov

- new non-Sudakov

- gluon pdfs are different
- effect of non-sing. terms visible

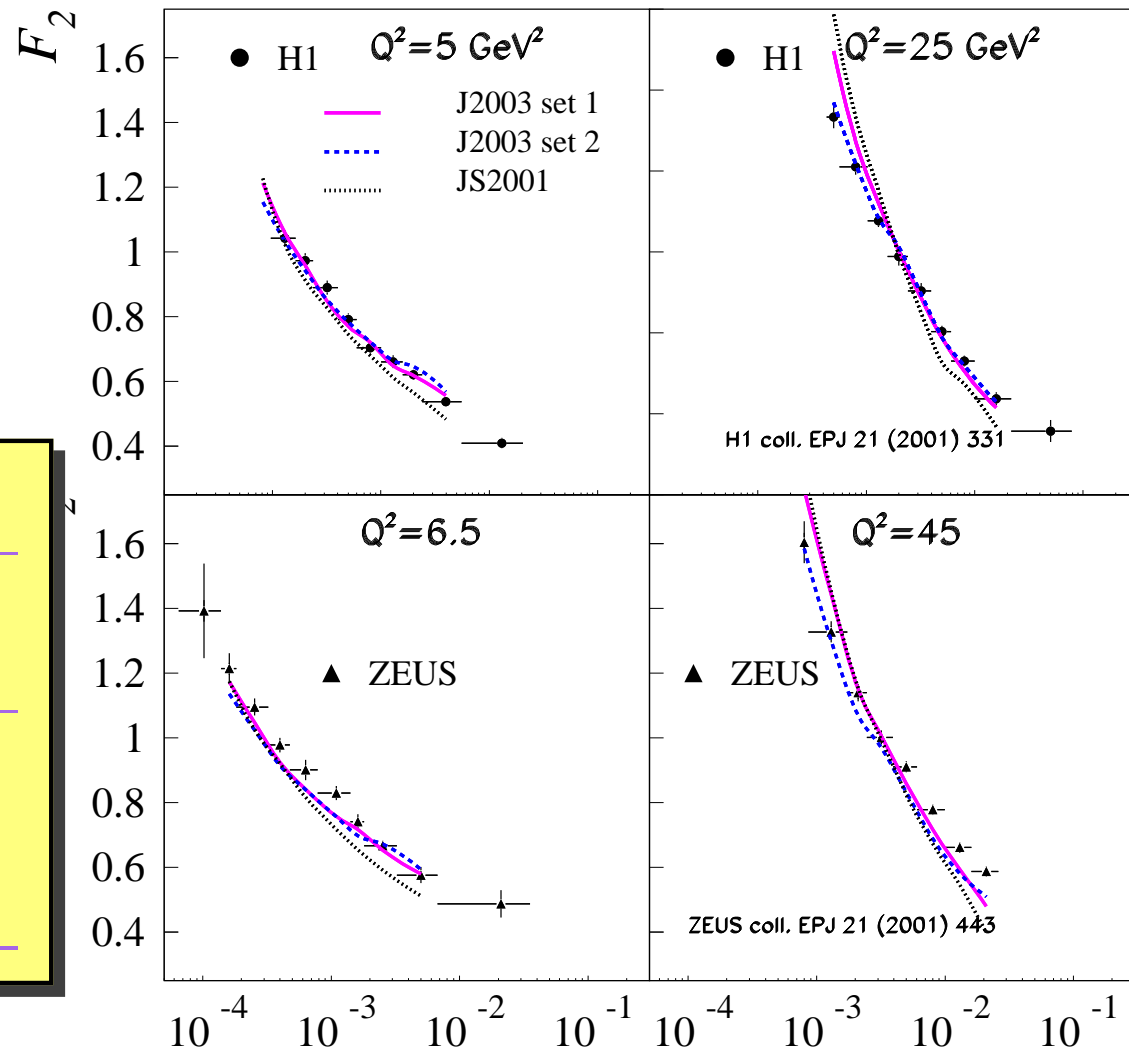


Precision fits to $F_2(x, Q^2)$

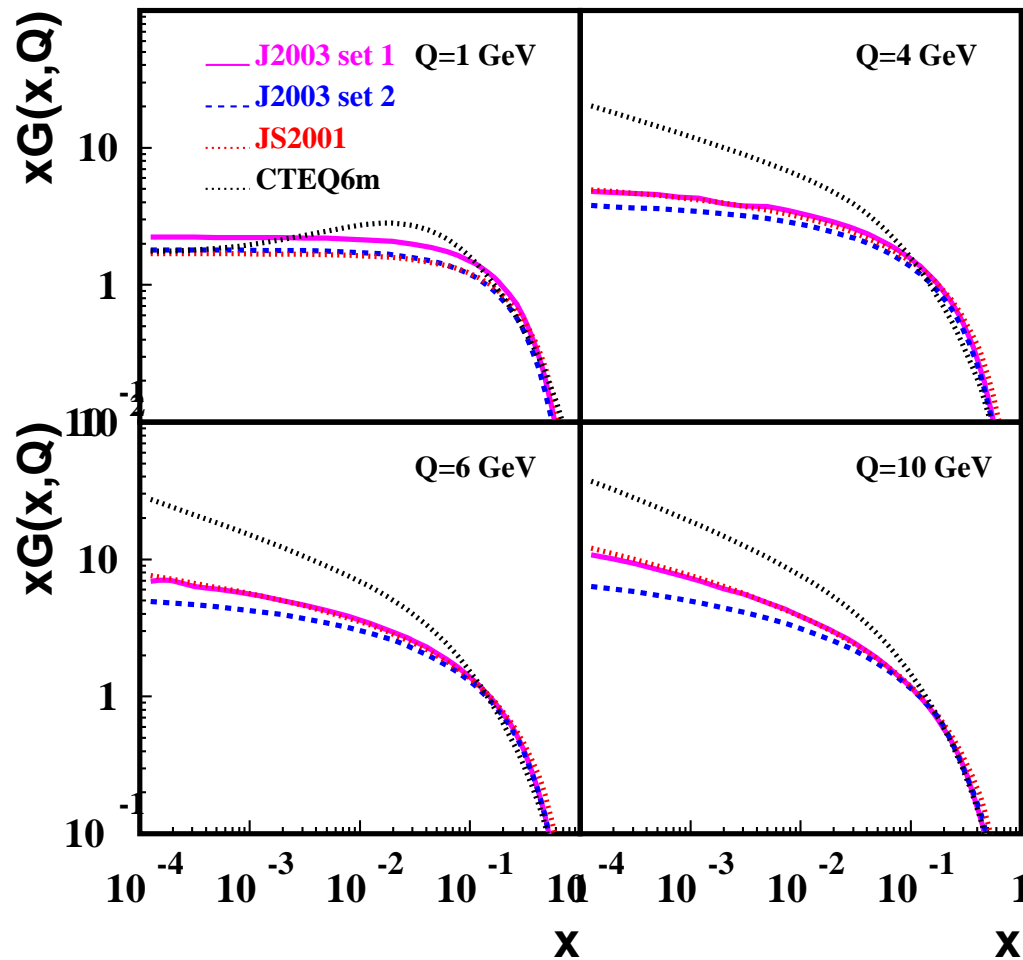
With $\sigma = \int dk_t^2 dx_g \mathcal{A}(x_g, k_t^2, \bar{q}) \sigma(\gamma^* g^* \rightarrow q\bar{q})$ fit $F_2(x, Q^2)$

- more precise data:
 - H1** NPB 470 (1996) 3., EPJ 21 (2001) 331.
 - ZEUS** ZPC 72 (1996) 399., EPJ 21 (2001) 443.
- fit $Q^2 > 4.5 \text{ GeV}^2, x < 0.005$
- small k_t - region ?
- full splitting function ?

Fits to $F_2(x, Q^2)$		
set	k_t^{cut} (GeV)	χ^2/ndf ndf = 248
$k_t^{cut} = Q_0$	1.33	1.29
full splitting	1.18	1.18
JS2001	0.25	4.8



CCFM unintegrated gluon density - integrated -



CCFM gluon integrated over k_t

$$\int_0^{\bar{q}} dk_t^2 x \mathcal{A}(x, k_t, \bar{q}) = xG(x, \bar{q})$$

CCFM gluon:

- at starting scale $q = 1$ GeV
flat !!!
- small x rise
generated perturbatively

DGLAP gluon:

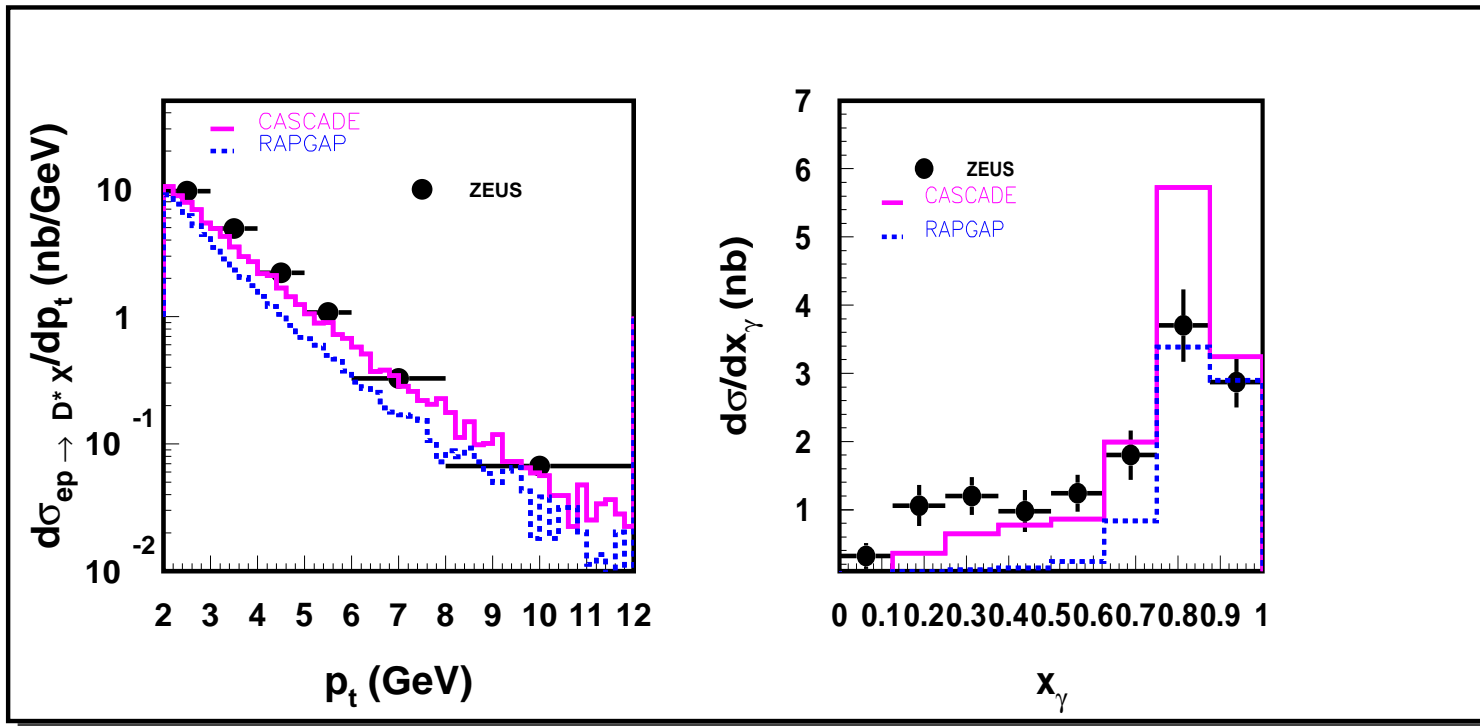
- steep rise at small x

Remember: gluon density is no observable, only cross sections !!!

Charm in γp

D^* photo - production (ZEUS Coll. EPJC 6 (1999) 67)

► p_t and η spectra well described in normalization and shape



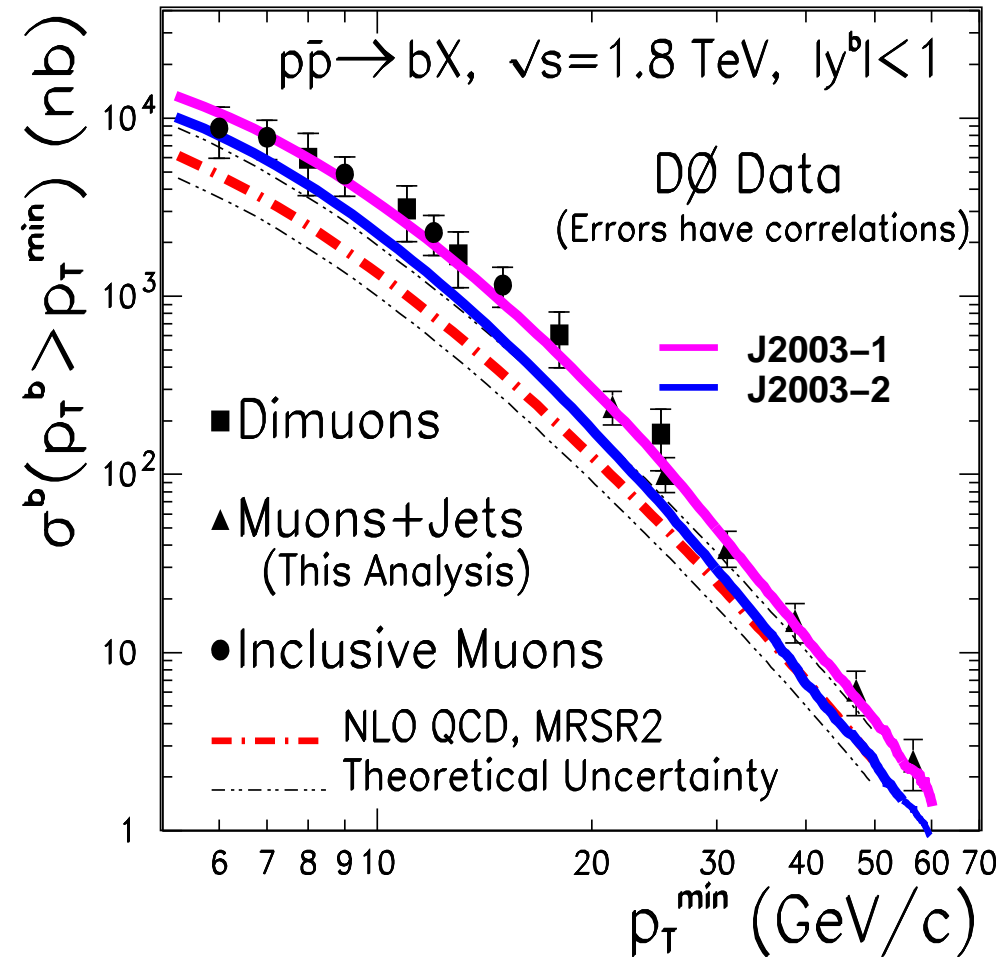
- k_t factorization includes already anomalous γ component
- no additional free parameters
- only $m_c = 1.5$ GeV and fragmentation function

Solution to the problem: $b\bar{b}$ production at Tevatron

Test universality of
unintegrated gluon density
from HERA

- ▶ use unintegrated gluon from F_2 fit at HERA
- ▶ use off-shell matrix element for $g^*g^* \rightarrow b\bar{b}$ with $m_b = 4.75$ GeV.
- ▶ set with singular terms ok
- ▶ even with full splitting fct ok

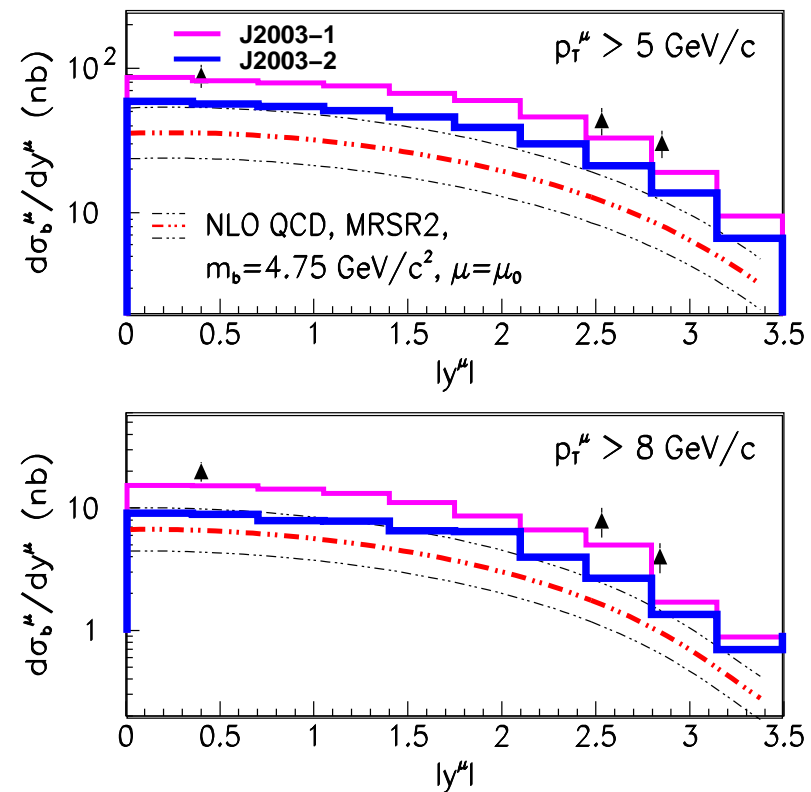
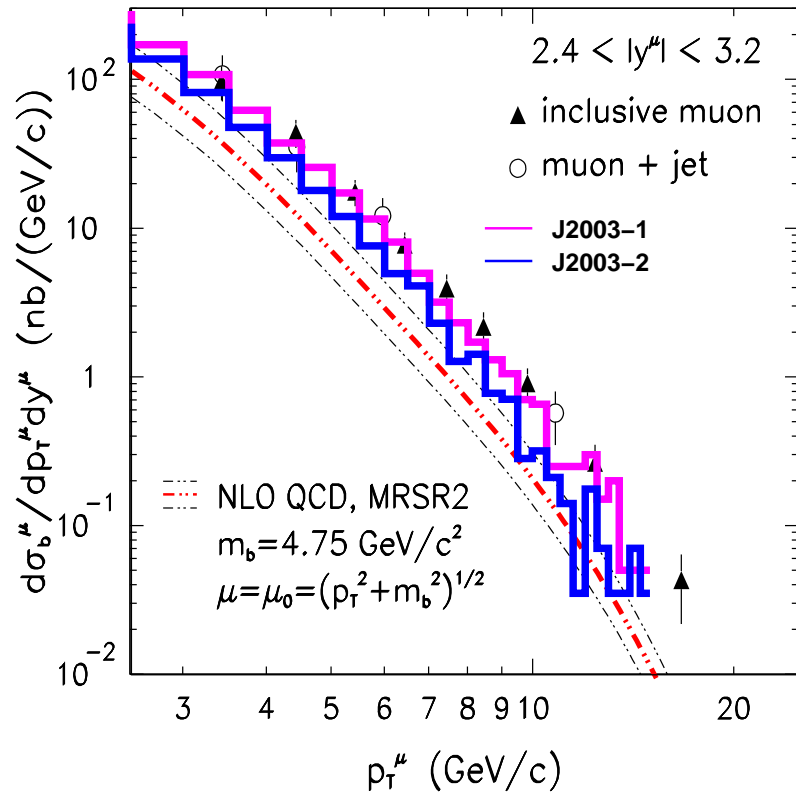
NOTE NLO off by factor 2



CASCADE w/o additional free parameters

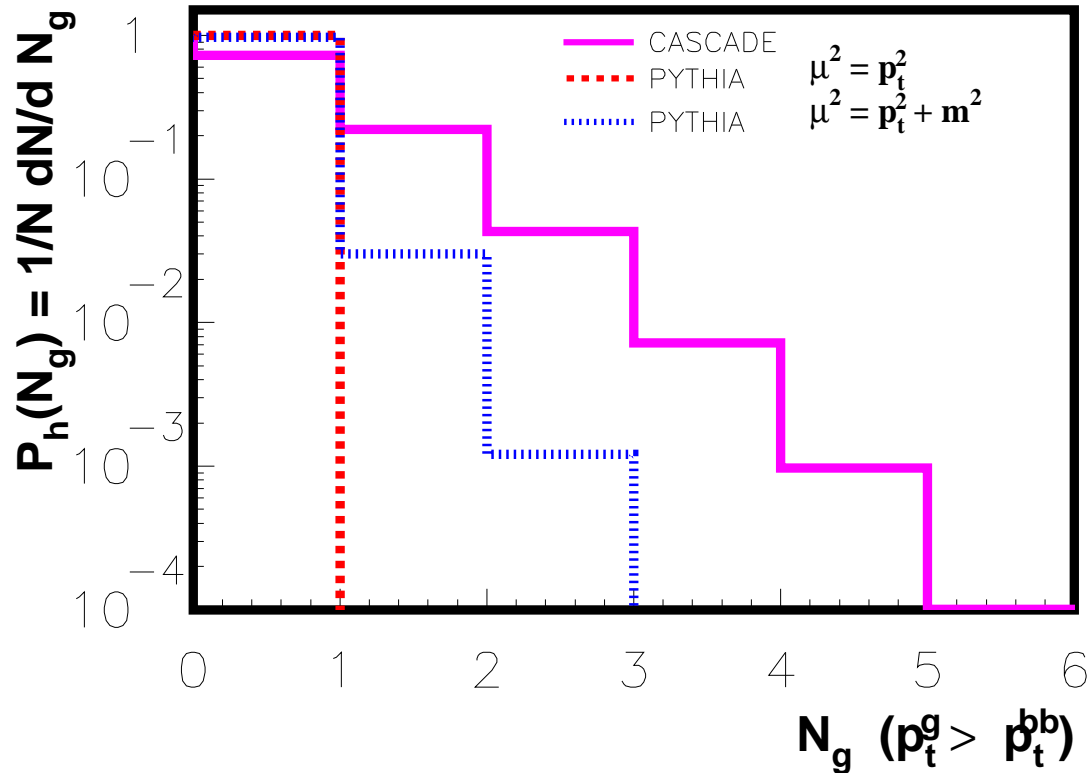
Solution to the problem: $b\bar{b}$ production at Tevatron

data from: D0 Collaboration B. Abbott et al., *Phy.Rev.Lett* 84 (2000) 5478



- CASCADE describes μ spectrum over huge range well
- NLO fails by factor ~ 2 (central) and ~ 4 (forward)

Why does k_t -factorization help for $b\bar{b}$ production at Tevatron



estimate higher order corrections

Nr of gluons with $p_t > p_t^{bb}$

LO: $\mathcal{O}(\alpha_s^2) \rightarrow N_g = 0$

NLO: $\mathcal{O}(\alpha_s^3) \rightarrow N_g = 1$

NNLO: $\mathcal{O}(\alpha_s^4) \rightarrow N_g = 2$

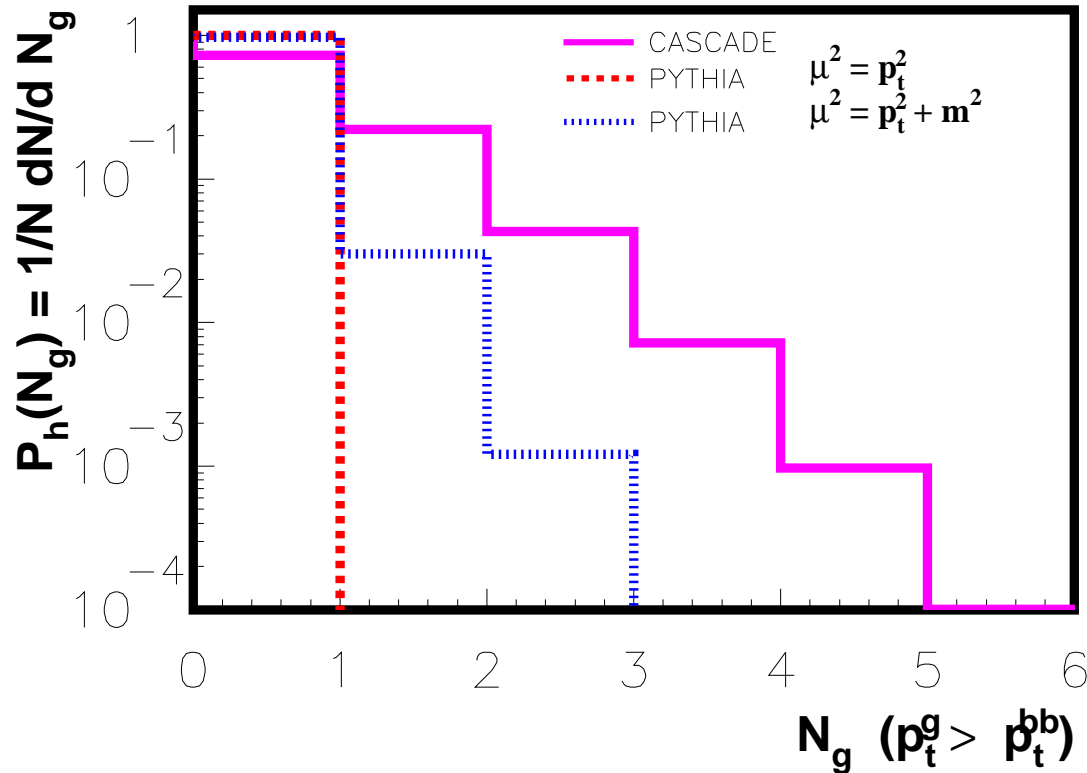
.....

.....

CASCADE $\rightarrow \mathcal{O}(\alpha_s^6)$

CASCADE with k_t factorization for estimation of higher order corrections

Why does k_t -factorization help for $b\bar{b}$ production at Tevatron



- recent calc using MC@NLO (Webber, Frixione et al.)
- parton shower in add to NLO increases Xsect (priv.comm.)
- now at 20 % level agreement with D0 data ...
- after years same result as CASCADE

.....
 CASCADE $\rightarrow \mathcal{O}(\alpha_s^6)$

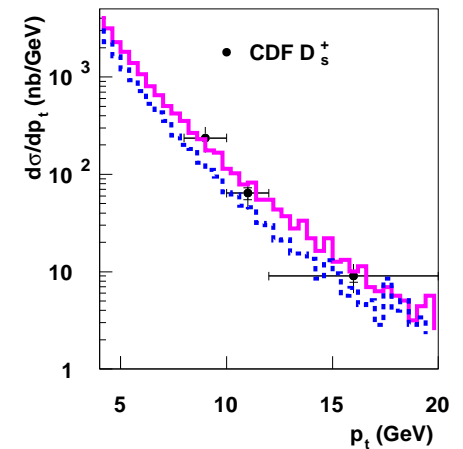
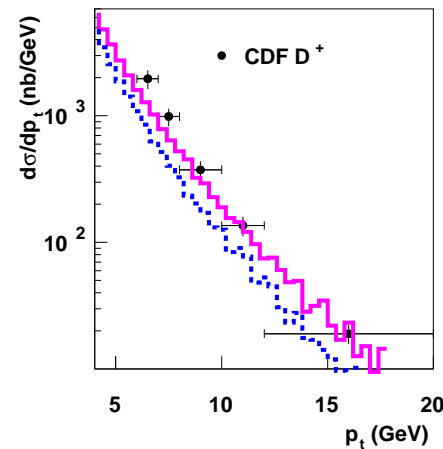
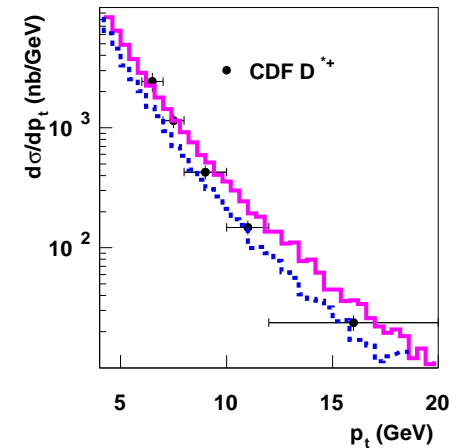
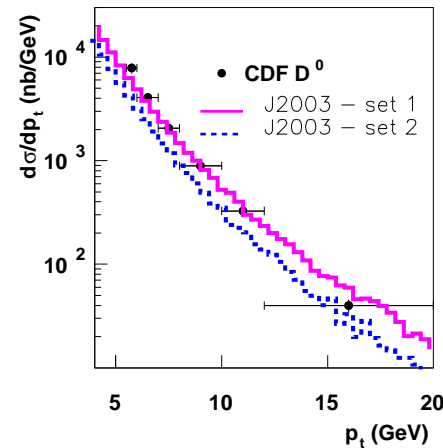
CASCADE with k_t factorization for estimation of higher order corrections

Solution to the problem: charm production at Tevatron

data from: CDF Collaboration D. Acosta et al., hep-ex/0307080

- measure charmed mesons in $|y| < 1$
- NLO by 100 - 50 % below data
- same model as for $b\bar{b}$
- similar problems as in $b\bar{b}$

- set with singular terms ok
- set with full splitting fct smaller but still larger than NLO

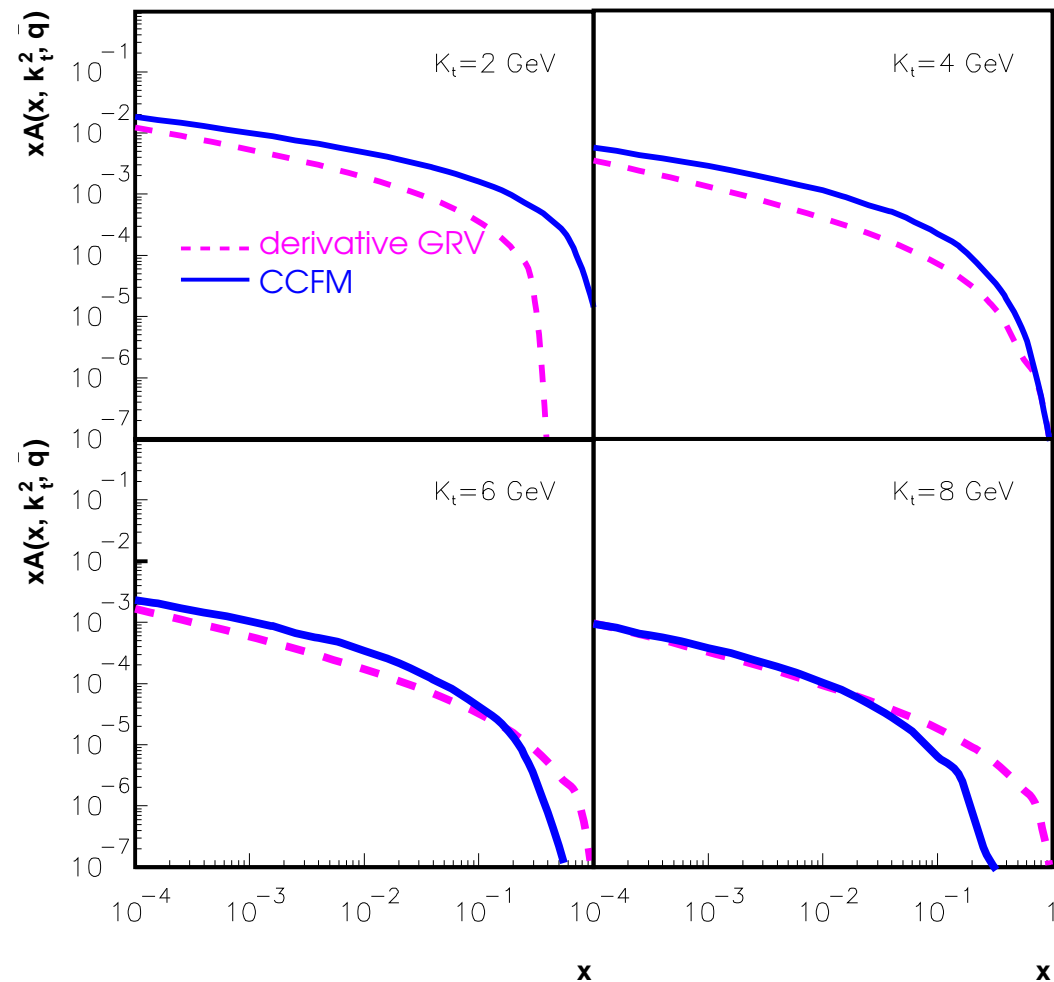


Success for CASCADE with k_t factorization !!!

Un-integrated Gluon Density of Photon

together with M. Hansson

- test machinery with one-loop (DGLAP)
- use gluon in photon from GRV as input
use normalization at input scale
- apply CCFM evolution (sing. terms only)
with parameters obtained from proton ($Q_0 = 1.4 \text{ GeV}$)



First un-integrated gluon density of real photon
with full CCFM evolution

$$\gamma\gamma \rightarrow b\bar{b}$$

together with M. Hansson

● use matrix elements in k_t - factorization

☞ $\gamma\gamma \rightarrow b\bar{b}$

☞ $\gamma g \rightarrow b\bar{b}$

☞ $gg \rightarrow b\bar{b}$

☞ universality...

● compare k_t -factorization & CCFM with NLO:

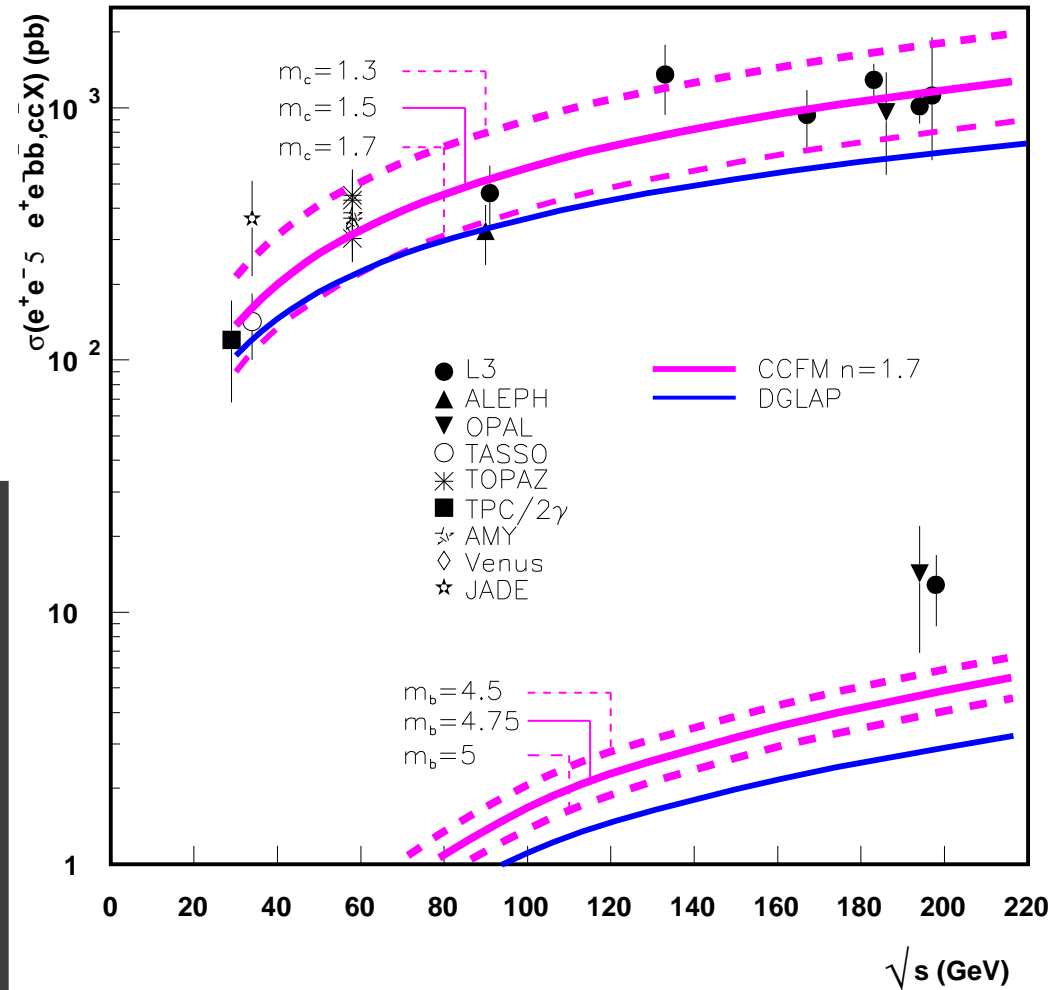
● using norm. from pdf

☞ CCFM similar to NLO

● determine norm. for gluon from charm ($n = 1.7$ for res. γ)

☞ CCFM larger than NLO

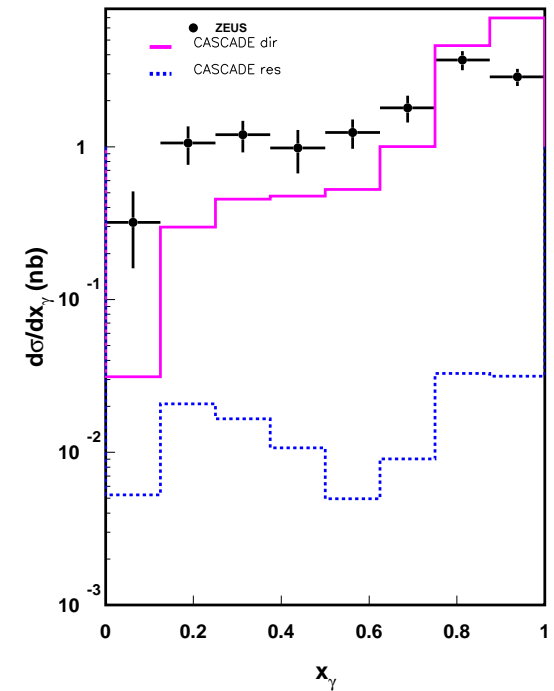
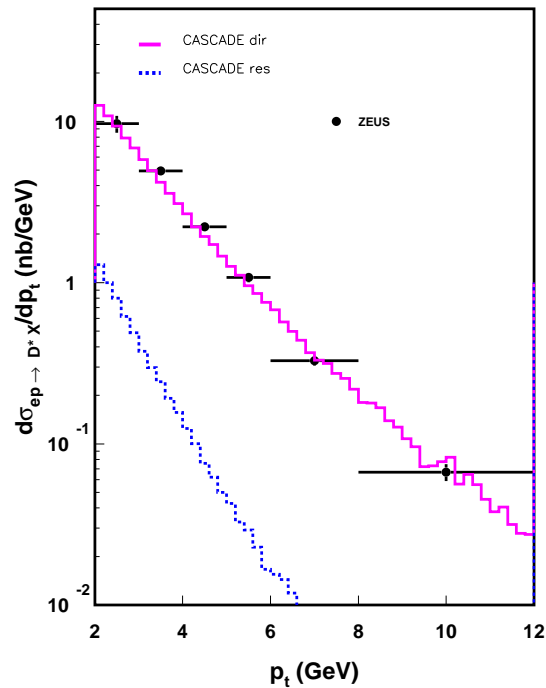
BUT still low for $\gamma\gamma \rightarrow b\bar{b}$



Charm in γp including resolved γ

D^* photo - production (ZEUS Coll. EPJC 6 (1999) 67)

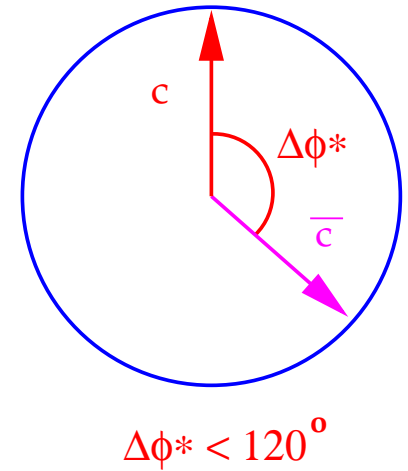
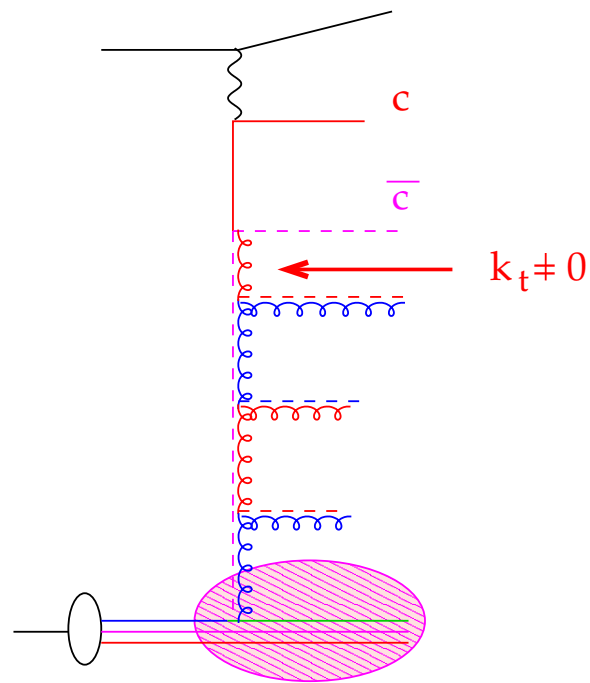
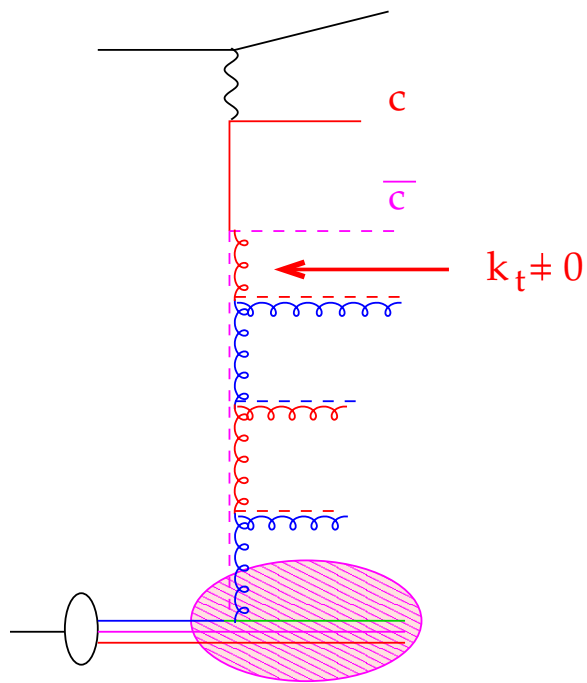
- use unintegrated gluon of γ
- use $gg \rightarrow Q\bar{Q}$



- in direct mode k_t factorization includes already anomalous γ component
- only small contribution from $gg \rightarrow Q\bar{Q}$

Unintegrated gluon density with heavy quarks

- identify both charm quarks
- reconstruct x_g
- reconstruct $\bar{q} = x_g \sqrt{s} \Xi$ with $\Xi = \exp(-2\eta^{c\bar{c}})$ in ep cms



● reconstruct

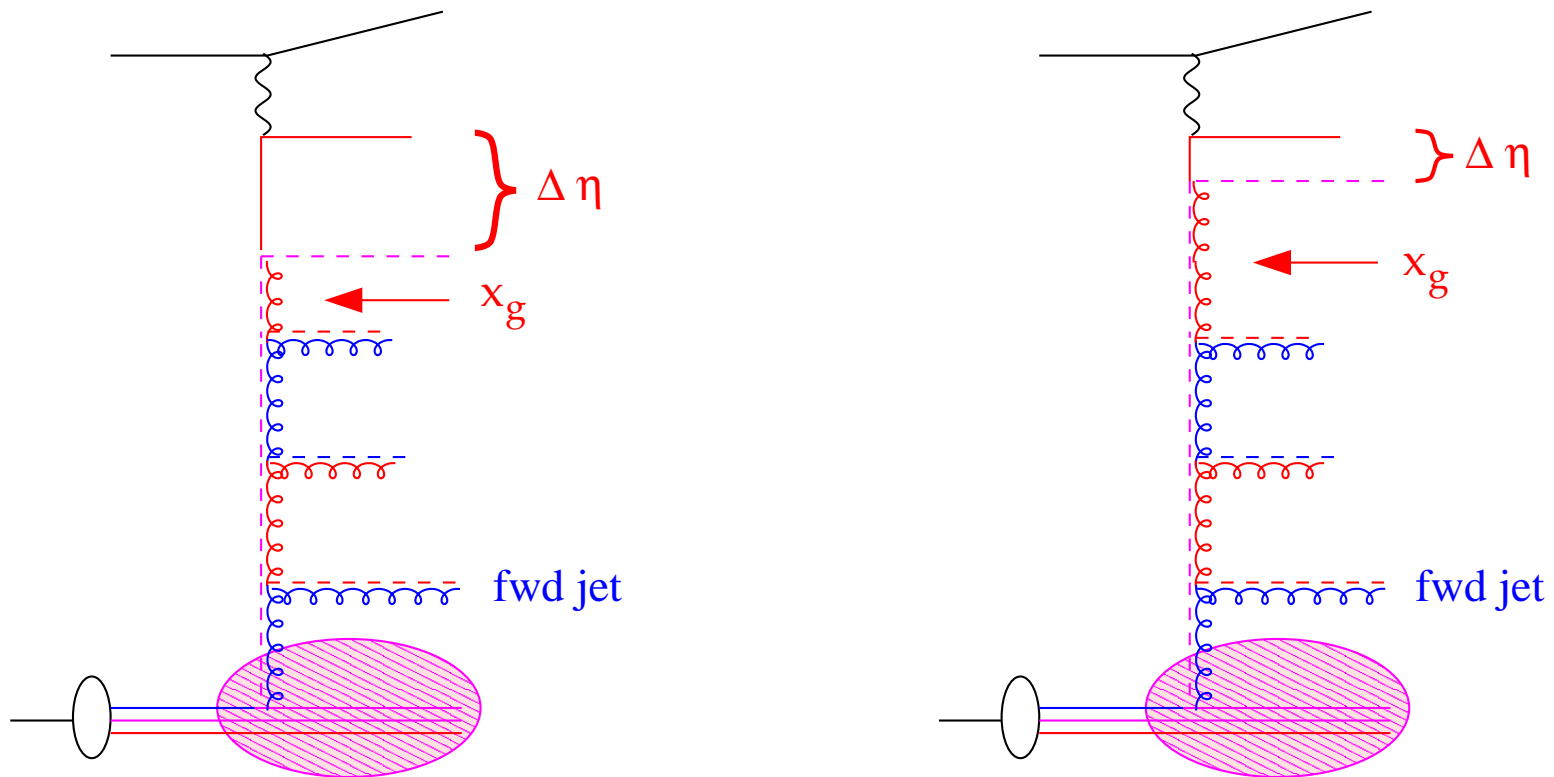
$$k_t = p_{\perp}^c + p_{\perp}^{\bar{c}} - p_{\perp}^{\gamma}$$

● reconstruct

$$S(x, Q^2, \Delta\Phi) = \frac{\int_0^{120^\circ} d\sigma d\Phi}{\int_0^{180^\circ} d\sigma d\Phi}$$

Forward jets and charm

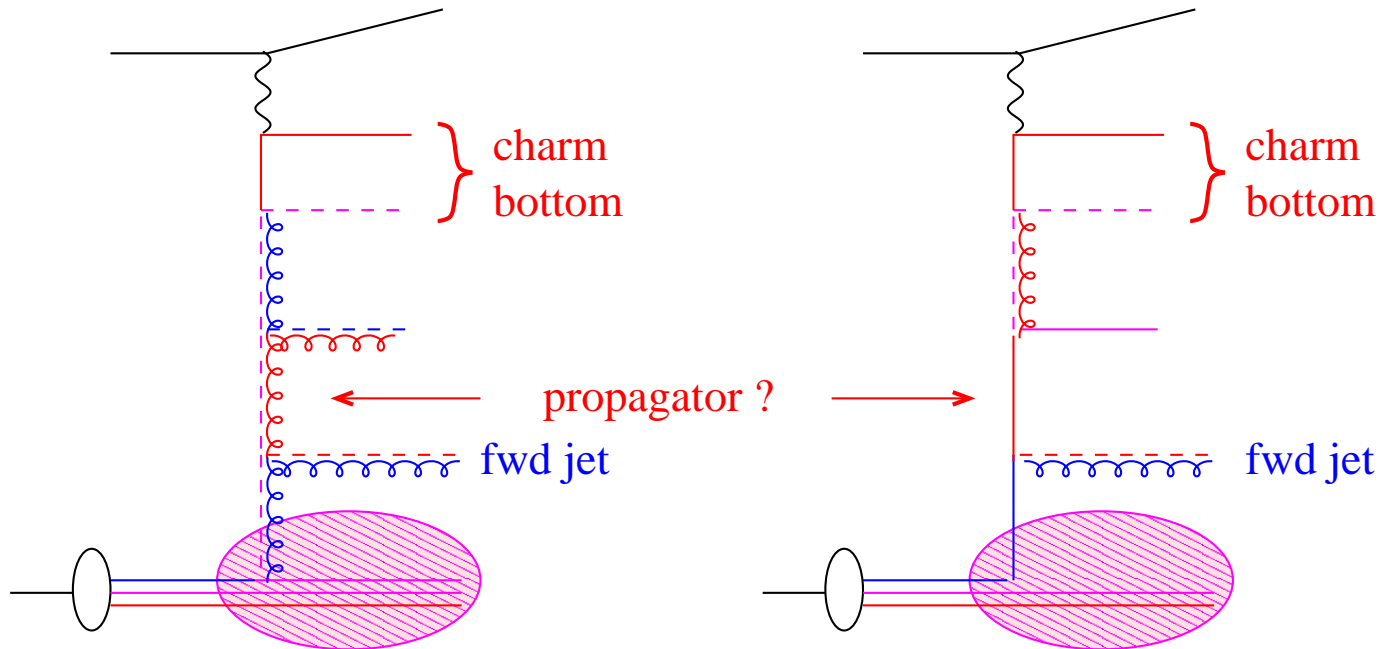
- identify both charm quarks, measure $\Delta\eta^{c\bar{c}}$, small $\Delta\eta^{c\bar{c}} \rightarrow$ small x_g
- measure $\Delta\eta^{c\bar{c},jet} = \eta^{c\bar{c}} - \eta_{fwd jet}$, small $\Delta\eta^{c\bar{c}}$, large $\Delta\eta^{c\bar{c},jet}$



investigate small x effects with heavy quarks...

Differentiate unintegrated density quarks and gluons

- identify one or both charm quarks reconst. 4 jets (incl. c-jets)
- measure $\cos \theta^*$... test propagator in initial state cascade !!!



- measure contribution of quark initiated charm production
- measure unintegrated quark density

The Beginning, Not the End

- k_t - factorization very successful
 - ✚ unintegrated gluon density from CCFM
 - ✚ precision fits to F_2 , including full splitting fct
 - ✚ describes measurements at HERA
 - ✚ even where collinear NLO fails
 - ✚ works also for charm and bottom in $p\bar{p}$
 - ✚ attempts also for bottom in $\gamma\gamma$
 - ✚ resummation to all orders helps
- investigate details of initial state radiation with heavy quarks
 - ✚ chance for very small x_g
 - ✚ new effects beyond DGLAP
 - ✚ unintegrated pdfs for quarks and gluons

The Beginning, Not the End

k_t - factorization and unintegrated pdfs

promising tool for calculation of higher orders

and correct event simulation at

HERA, Tevatron and **LHC**

Knowledge of unintegrated pdfs

important for Higgs production at

LHC

$b\bar{b}$ production in DIS at HERA: H1 and ZEUS

H1(prel.)

$$2 < Q^2 < 100 \text{ GeV}^2, 0.1 < y < 0.8,$$

$$p_t^\mu > 2 \text{ GeV}, 35^\circ < \theta^\mu < 130^\circ$$

visible x-section $ep \rightarrow e' b\bar{b}X \rightarrow \mu X$:

$$\sigma = 39 \pm 8(\text{stat.}) \pm 10(\text{syst.}) \text{ pb}$$

$$\text{NLO: } \sigma = 11 \pm 2 \text{ pb}$$

$$\text{CASCADE } \sigma(ep \rightarrow e' b\bar{b}X) = 15 \text{ pb}$$

$$R_{MC}(\text{H1}) = \frac{\sigma_{\text{measured}}}{\sigma_{MC}} = 2.6$$

ZEUS(prel.) ICHEP 2002

$$Q^2 > 2 \text{ GeV}^2, 0.05 < y < 0.7,$$

$$E_{T,jet}^{Breit} > 6 \text{ GeV}, -2 < \eta_{jet}^{lab} < 2.5$$

$$p^\mu > 2 \text{ GeV}, 30^\circ < \theta^\mu < 160^\circ$$

x-section $ep \rightarrow e' b\bar{b}X \rightarrow e' jet \mu X$:

$$\sigma = 38.7 \pm 7.7(\text{stat.})_{5.0}^{6.1}(\text{syst.}) \text{ pb}$$

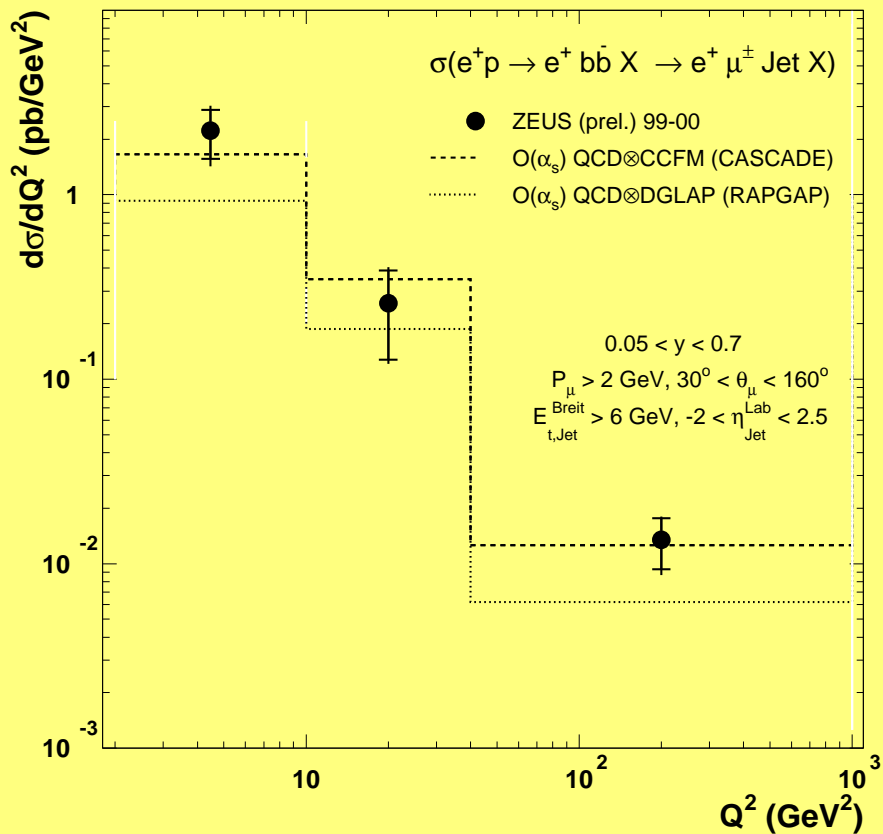
$$\text{NLO: } \sigma = 28.1 \pm 2 \text{ pb}$$

$$\text{CASCADE } \sigma(ep \rightarrow e' b\bar{b}X) = 35 \text{ pb}$$

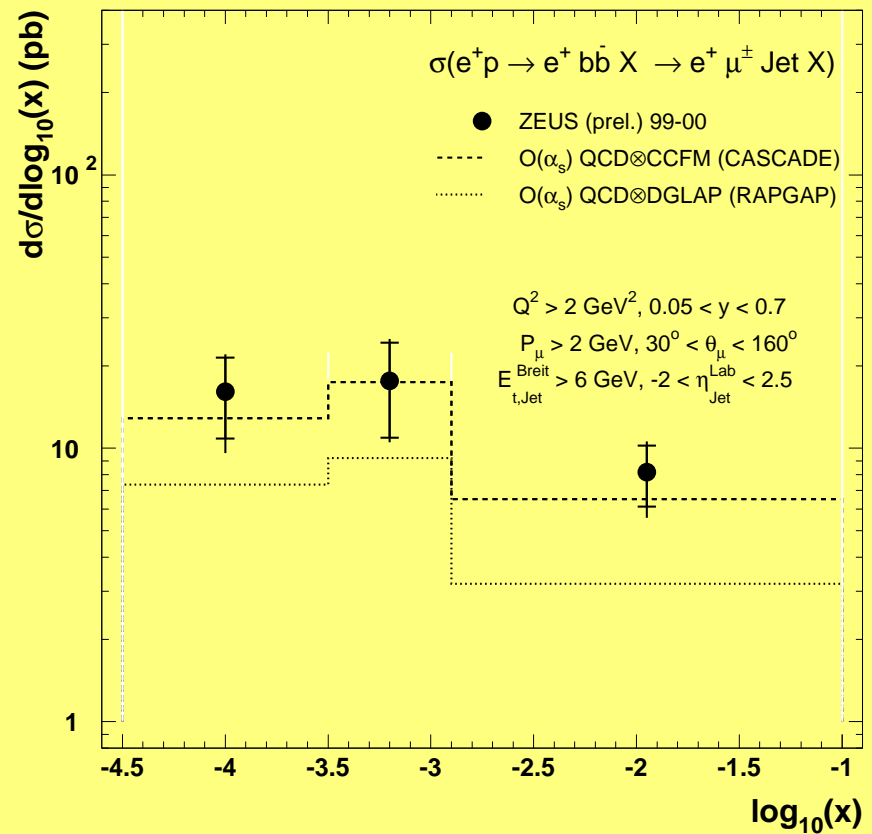
$$R_{MC}(\text{ZEUS}) = \frac{\sigma_{\text{measured}}}{\sigma_{MC}} = 1.1$$

$b\bar{b}$ production in DIS at HERA: ZEUS

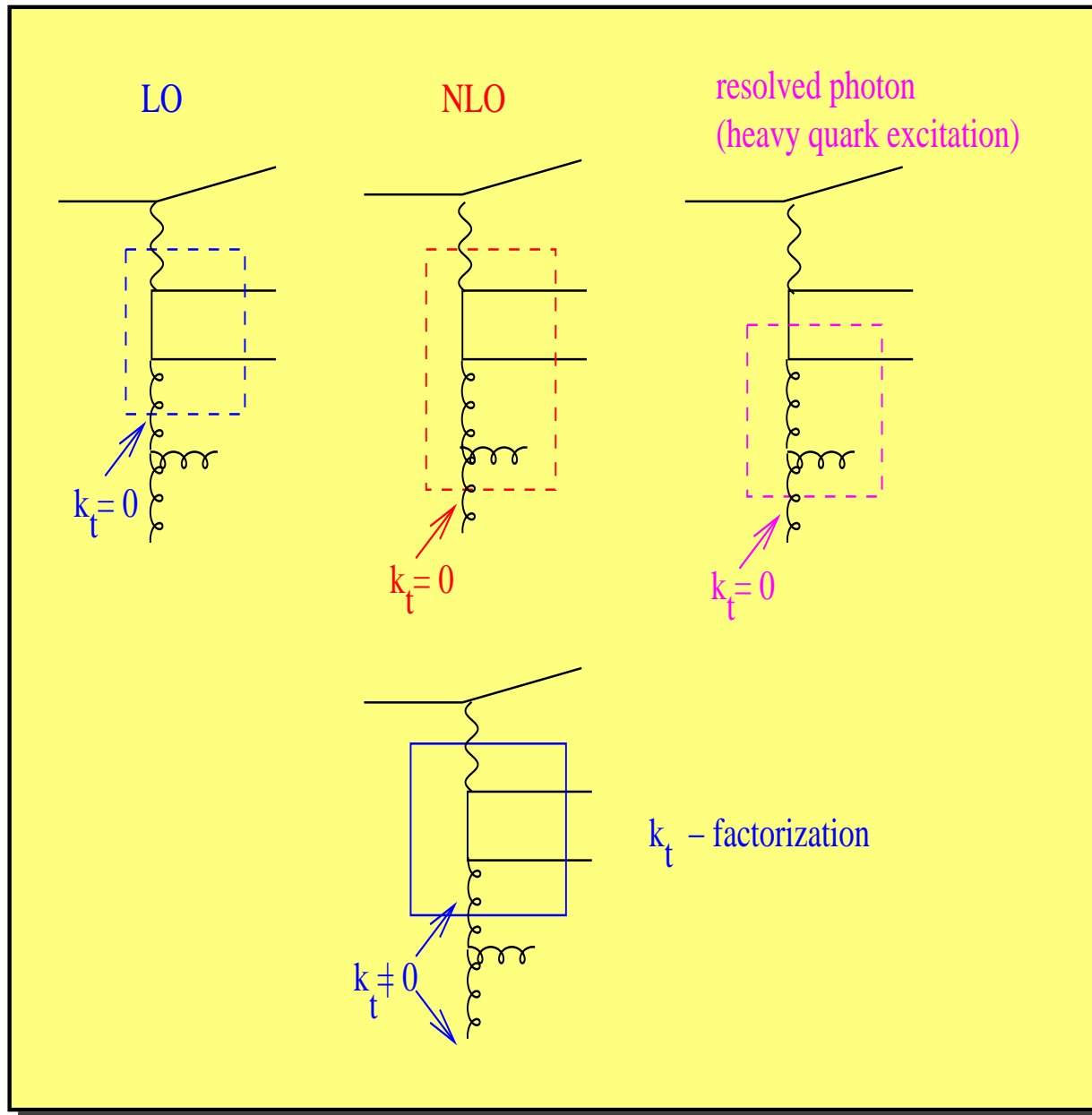
ZEUS



ZEUS



Resolved - γ , NLO and k_t - factorization



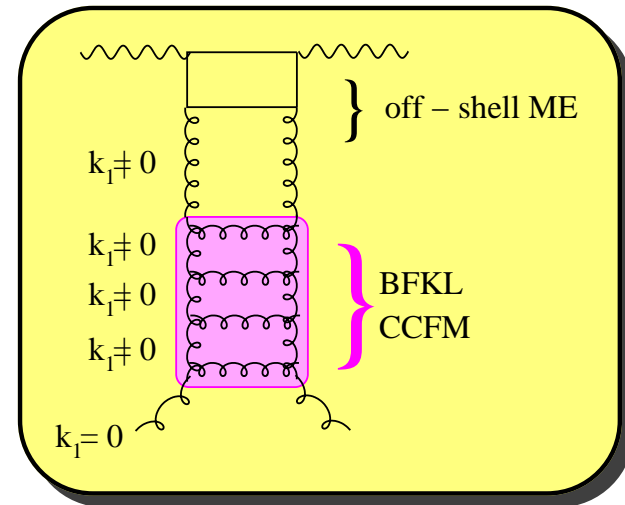
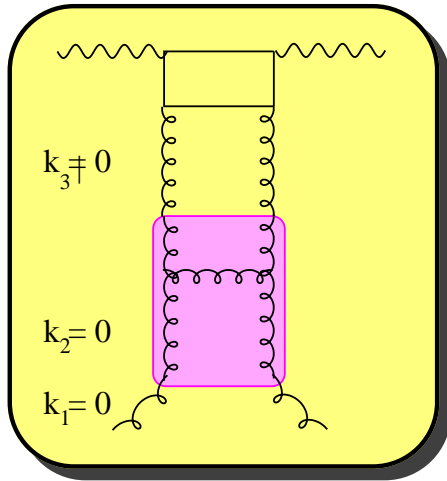
k_t factorization

- **NLO corrections**
- **anomalous γ**
- **even in NLO**
- **includes NNLO**
- **includes NNNLO**
- **includes NNNNLO**

k_t factorization has

- no problem with:**
- **negative weights....**
 - **matching to PS**
 - **matching to hadronisation**

k_t - and collinear factorization



off - shell matrix element

- 1-loop correction to Born approximation
- high energy limit of NLO !!!

$$\sigma = \int \frac{dz}{z} d^2 k_t \hat{\sigma}\left(\frac{x}{z}, k_t\right) \mathcal{F}(z, k_t)$$

$$\mathcal{F}(z, k_t) = \int \frac{dz}{z} \tilde{\mathcal{F}}(x/z, k_t; Q_0) \bar{f}_0(z, Q_0)$$

factorize k_t dependence in \mathcal{F} and insert in σ

- improved coefficient and splitting functions to all order