Determination of $\alpha_s$ from Inclusive Jets and Dijets at HERA using the ZEUS Detector

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A thesis submitted to the Faculty of Graduate Studies
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Graduate Program in Physics and Astronomy
York University
Toronto, Ontario, CANADA

July 2006
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Abstract

Inclusive jet cross sections for neutral current deep inelastic scattering processes $e^\pm p \to e^\pm X + jet(s)$ for boson virtualities in the range $Q^2 > 125$ GeV$^2$ have been measured by the ZEUS detector at HERA using data collected between 1998 and 2000, which corresponds to an integrated luminosity of 82.2 pb$^{-1}$. Inclusive event and dijet cross sections have also been measured for the same data set in the range $Q^2 > 125$ GeV$^2$. Jets were identified in the Breit frame using the longitudinally invariant $k_T$-cluster algorithm. There are four sets of measurements presented: inclusive jet differential cross sections with respect to $Q^2$, $E_{T,jet}^B$ and $\eta^B$; double differential cross sections with respect to $E_{T,jet}^B$ and $Q^2$; an inclusive event differential cross section with respect to $Q^2$, and dijet differential cross sections with respect to $Q^2$ and $E_{T,jet}^B$ for each jet.

These measurements were compared to next-to-leading order calculations and within the experimental and theoretical uncertainties, reasonable agreement is shown. This comparison was used to extract values for the QCD parameter, $\alpha_s(M_Z)$. For the analysis involving inclusive jets, the values obtained were

$$\alpha_s(M_Z) = 0.1196 \pm 0.0010 \ (stat.) + 0.0020 \ (syst.) \ + 0.0034 \ (th.) - 0.0014 \ (syst.) \ (th.) \ \text{for} \ Q^2 > 500 \text{ GeV}^2,$$

$$\alpha_s(M_Z) = 0.1198 \pm 0.0008 \ (stat.) + 0.0035 \ (syst.) + 0.0033 \ (th.) - 0.0021 \ (th.) \ \text{for} \ E_{T,jet}^B > 14 \text{ GeV}.$$

For the analysis involving the ratio of differential cross sections with respect to $Q^2$ between exclusive dijet events and inclusive events the value obtained was

$$\alpha_s(M_Z) = 0.1173 \pm 0.0012 \ (stat.) + 0.0026 \ (syst.) + 0.0048 \ (th.) - 0.0043 \ (syst.) - 0.0043 \ (th.) \ \text{for} \ Q^2 > 500 \text{ GeV}^2.$$


Acknowledgements

DESY is a large and complex operation, from the HERA accelerator to the ZEUS experiment itself. Without the dedication and hard work of many people, none of the large amount of progress made in the field of particle physics, including this thesis and the results presented here, would be possible.

In particular, I would like to thank my supervisor, Sampa Bhadra for providing me with the opportunity and funding to study in this field. Her endless enthusiasm and guidance have been invaluable during this period. I would like to thank Thomas Schörner-Sadenius for doing my second analysis. His partnership and assistance over the months has also been much appreciated. I would like to thank the coordinators of the QCD group, namely Claudia Glasman, Juan Terron, Sergei Chekanov and Alexandre Savin. Without their expertise and insight, this study would have been much more difficult. I would also like to thank the other senior members of the Canada Group; Scott Menary, Mara Soares, Roberval Walsh and Sergei Fourletov for the help and advice during my time at DESY, as well as Marlene Caplan at York for her help on administrative matters.

Also, I would like to mention the friends I have made during my time in Hamburg. In particular, Andreas Ochs and Raphael Galea for making me feel welcome when I first arrived at DESY, Ying Cui, who has kept me sane, and the other members of the Canada Group; Thorsten Koop, Jerome Whyte and Umer Noor, who have all kept me entertained.
Most importantly, I would like to thank my parents. In particular, my father who passed away during the completion of this thesis. He is constantly in my thoughts. Without their unwavering dedication and support, I would not be anywhere close to where I am today.
Contributions to the ZEUS Experiment

The completion of this thesis marks my seventh year as a member of the Canadian section of the ZEUS collaboration, during which I have completed my Masters Degree and now my Ph. D. It has been a highly rewarding and stimulating period which has provided me with the opportunity to learn about the world of High Energy Physics, in particular, how a large physics accelerator complex with all its various detectors and components works, as well as how to carry out physics analyses, including aspects such as jet physics and deep inelastic scattering collisions, not to mention all the theoretical knowledge I have gained.

From mid 1999 to 2001, I worked as part of the York University group that assisted in the construction of the new Straw Tube Tracker (STT) detectors which were installed into the ZEUS detector as part of the major HERA upgrade. This entailed building and testing the individual straw sectors before they were shipped from York University to Hamburg to be installed into ZEUS.

In 2002 I moved to DESY permanently and became a member of the ZEUS Heavy Flavour (HFL) and QCD groups. The analyses presented in this thesis were carried out as part of those groups. I also joined the Offline group and became the software manager from March 2003 until August 2004. In this role, my tasks included debugging code and updating software libraries for different computer platforms that are used for ZEUS analyses. From 2002 to 2006 I did regular shifts in the ZEUS control room as deputy shift
leader. This involved monitoring ZEUS components and data acquisition, as well as doing safety inspection rounds. Since 2003, I have also been the Canada group system administrator.

The analyses presented in this thesis were started in 2003 and based on data taken by the ZEUS detector during the years 1998 to 2000. The result were presented to the “13TH INTERNATIONAL WORKSHOP ON DEEP INELASTIC SCATTERING” (DIS2005) in Madison, Wisconsin, as well as at the CAP2005 conference in Vancouver, B.C.

The outcome from these analyses is a measurement of the QCD parameter $\alpha_s$ to a higher overall precision (i.e. taking into consideration the experimental and theoretical uncertainties) than any measurement made at HERA to date and one that is competitive with the current world average. The importance of this parameter is twofold. Firstly it is the parameter of perturbative QCD. It cannot yet be predicted by theory and hence has to be measured. Consequently, there is no limit to the precision with which we would like to measure it. Should a theory eventually arise that predicts the value of $\alpha_s$, its validity will be judged on the comparison of that theory with the most precise measurements. Secondly, the value of this parameter is required as an input to current theories; not only QCD which is part of the Standard Model, but also more exotic theories such as Supersymmetry, String Theory and Grand Unified Theories.
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1 Introduction

1.1 The Standard Model

Scientists have sought answers to two fundamental questions; what are the constituents of the material that makes up the world around us and how do those constituents interact with each other? The Standard Model is the most successful theory that physicists have found that provides an answer to those questions. It states that matter is made up of twelve fundamental fermions, known as quarks and leptons. These particles and some of their properties are summarized in Table 1.01. They interact with each other via four different forces, namely the strong force, the electromagnetic force, weak force and the gravitational force. The latter, being by far the weakest of the four, is only significant on a large (planetary) scale and is not regarded as relevant to particle physics. The mediation of these forces between the fundamental particles is done by means of exchange bosons, also shown in Table 1.01. The weak force is mediated by the vector bosons, $W^\pm$ and $Z^0$, the electromagnetic force that acts between charged particles is mediated by the photon and the strong force that acts on coloured particles is mediated by the gluon.

The electromagnetic and weak forces have been unified into one electroweak force. It is the aim of theoretical physicists to unify all forces into one Grand Unified Theory. Much progress has been made towards this goal, but it is gravity, although very well described by General Relativity, that is proving hardest to incorporate.
### Table 1.01: Quarks, leptons and bosons of the Standard Model.

<table>
<thead>
<tr>
<th>Fermions (spin (1/2))</th>
<th>Quarks</th>
<th>Leptons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Particle</strong></td>
<td><strong>Symbol</strong></td>
<td><strong>Interactions</strong></td>
</tr>
<tr>
<td>Down</td>
<td>(d)</td>
<td>Strong, EM, Weak</td>
</tr>
<tr>
<td>Up</td>
<td>(u)</td>
<td>Strong, EM, Weak</td>
</tr>
<tr>
<td>Strange</td>
<td>(s)</td>
<td>Strong, EM, Weak</td>
</tr>
<tr>
<td>Charm</td>
<td>(c)</td>
<td>Strong, EM, Weak</td>
</tr>
<tr>
<td>Beauty</td>
<td>(b)</td>
<td>Strong, EM, Weak</td>
</tr>
<tr>
<td>Top</td>
<td>(t)</td>
<td>Strong, EM, Weak</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bosons (spin 1)</th>
<th>Particle</th>
<th>Symbol</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon</td>
<td>(\gamma)</td>
<td>EM</td>
<td></td>
</tr>
<tr>
<td>(W, Z)</td>
<td>(W^\pm, Z^0)</td>
<td>Weak</td>
<td></td>
</tr>
<tr>
<td>Gluon</td>
<td>(g)</td>
<td>Strong</td>
<td></td>
</tr>
</tbody>
</table>

Scattering processes occur through the exchange of one of the bosons in Table 1.01 (corresponding to the force that controls the interaction) between a projectile and a target. In the case of the electromagnetic force, this is a photon. The four-momentum, \(q\), of this photon dictates the resolution at which the target is ‘seen’. For example, Rutherford scattering proceeded via the use of alpha particles (helium nuclei), which were aimed at a gold nucleus target. Naturally occurring alpha particles, as used by Rutherford, are relatively low in energy (a few MeV) and so the resolution at which they interact with the gold nuclei is low. The nucleus is seen as a single particle and the individual nucleons cannot be distinguished. For high energy scattering experiments that use energies in the
GeV to TeV range, the four-momentum of the exchanged photon is far higher. For this reason, the individual constituents of the target can be resolved. In the case of electron-proton scattering, done with centre-of-mass energies of several hundred GeV, deep inelastic scattering\(^1\) (DIS) interactions occur. These are able to achieve resolutions of better than \(10^{-18}\) m and therefore the quarks in the proton can be resolved. This has allowed the structure of the hadrons to be established and led to the Quark Parton Model of the proton that is described in Chapter 2.

Quantum ElectroDynamics (QED) is based on quantum field theory and provides a complete relativistic quantum mechanical picture of how a charged particle interacts with, and couples to, an electromagnetic field via a single gauge boson (photon). It is an Abelian theory based on the \(U(1)\) symmetry group, which is linked to the fact that the photon carries no charge and therefore cannot interact directly with itself. It is based on the Lagrangian density:

\[
L_{QED} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} e^{\gamma^\mu} i [\partial_\mu + ieA_\mu] \psi - m_e \bar{\psi} \psi e. \tag{1.01}
\]

The first term describes the electromagnetic field defined by the four-potential, \(A^\mu \equiv (\phi, \vec{A})\) where \(\phi\) is the scalar Coulomb potential and \(\vec{A}\) is the vector potential. The electromagnetic field tensor is given by \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\). Using the Euler-Lagrange equations corresponding to this term, the Maxwell equations are obtained that describe electromagnetic waves. The last term describes the free electron of mass \(m_e\) at rest. This is represented by a four component Dirac spinor, \(\psi_e\). The first part of the middle term

\[\text{Deep Inelastic Scattering, ‘Deep’ refers to the high resolution at which the target is probed and Inelastic refers to the fact that the target is destroyed as a single entity.}\]
represents the kinetic energy of the free electron and the second part of that term represents the interaction of the electron with the electromagnetic field. QED processes are involved in electron-proton (ep) scattering, due to the emission of a mediating photon from the electron that couples to a quark in the proton.

### 1.1.1 Quantum ChromoDynamics

Quantum ChromoDynamics (QCD) describes the strong force that exists between particles with a colour charge, namely the quarks and gluons that make up the hadrons (baryons and mesons) such as the proton. No particle with overall colour has ever been observed in nature. Baryons consist of three quarks bound together by gluons; one quark of each possible colour charge, Red(R), Green(G) and Blue(B), making an overall colourless particle. Mesons consist of a quark and an antiquark (of the opposite colour charge) bound together and are likewise overall colourless. The colour quantum number is also necessary to explain how hadrons consisting of three identical quarks can exist. The Ω⁻ particle is a hadron that consists of three strange quarks, sss. According to the Pauli Exclusion principle, the existence of two or more quarks with identical quantum number in the same particle is forbidden. Through the addition of the colour quantum number to the flavour quantum number for quarks, the existence of this particle can be explained.

Experiments have shown that the quarks in hadrons only account for about 50% of the momentum of the particle. The rest is carried by the gluons that bind the quarks together. These gluons are continuously exchanged between these three ‘valence’ quarks. However, the picture is more complicated than that. These gluons can split into quark-
antiquark pairs in the fleeting period between when they are emitted and reabsorbed. In
turn these ‘sea’ quarks can emit gluons, which in turn can split into quark-antiquark pairs
and so on. The result is that the baryon is not simply three quarks bound together by
gluons, but a seething mass of quarks and gluons that looks increasingly complicated as it
is probed at finer and finer distance scales.

QCD, also based on quantum field theory, describes the strong interaction in the same
way that QED describes electromagnetic interaction. It is a non-Abelian gauge field
theory based on the $SU(3)_C$ symmetry group that describes the interactions between
coloured objects and is based on the Lagrangian density (up to gauge-fixing terms):

$$L_{QCD} = -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + \sum_q \bar{\psi}_q^i (i D_{ij} - m_q) \psi_j^i ,$$

where the sum on $q$ is over the 6 quark flavours ($u, d, s, c, b, t$), the gluon tensor is given
by

$$F_{\mu\nu}^{(a)} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + g_s f^{abc} A_{\mu}^{b} A_{\nu}^{c}$$

and the gauge covariant derivative is given by

$$(D_{\mu})_{ij} = \delta^i_j \partial_{\mu} - ig_s A_{\mu}^{a} T_{ij}^{a} .$$

The $T^a$'s are the $SU(3)$ representation matrices (eight Gell-Mann matrices), which have
been normalized such that

$$tr(T^a T^b) = \frac{\delta^{ab}}{2} .$$

The $f^{abc}$ are the structure constants of the $SU(3)$ group. The $\psi_q^i$ are the four component
Dirac spinors associated with each quark field of colour $i$ and flavour $q$ and the $A$’s are
the gluon fields. The third term on the right hand side of equation 1.03 leads to the non-
Abelian nature of QCD that gives rise to the self-interaction of the gluon. In other words,
the gluon itself carries a colour charge, hence it can couple to other gluons. This is contrary to QED, in which the photon carries no electric charge and so there is no such self-interaction. These gluon self-interactions led to the concept of asymptotic freedom, a feature of QCD that is absent in QED. The coupling strength in Equation 1.04 is given by $g_s$. However the strong coupling constant $\alpha_s$ is more commonly used and is given by

$$\alpha_s = \frac{g_s^2}{4\pi}. \quad (1.06)$$

This is analogous to the fine structure constant, $\alpha$, used to represent the coupling strength in QED. The value of the strong coupling constant$^2$, $\alpha_s$, is not predicted in QCD and so it has to be measured experimentally. \textit{This measurement is one of the principal aims of this thesis.} When $\alpha_s$ is small, it is possible to use the perturbative approach to perform calculations in the same manner as QED calculations are done. Perturbative QCD (pQCD) has been used for the last 30 years to make predictions and has had considerable success. These theoretical calculations can be compared to experimental measurements as a test of pQCD.

\subsection*{1.1.2 Asymptotic Freedom and Confinement}

The self-coupling of the gluon gives rise to two important features of the strong force; \textit{asymptotic freedom} and \textit{confinement}. The strong force between two coloured particles is similar to the force provided by an elastic band between two objects. Unlike the electromagnetic force, the strong force gets stronger with distance, so as the two particles

$^2$ The term coupling constant is somewhat of a misnomer given that it is not constant but varies with scale.
are separated, the force between them increases. The relationship between distance and strength of the strong force is shown in Figure 1.01. If the distance increases sufficiently, the potential energy stored in the system is great enough for it to become energetically favourable for the force lines between the particles (represented by the elastic band) to ‘snap’ and a quark-antiquark pair (of the same colour-anticolour) to be released from the vacuum. The rest mass of these particles comes from the (now reduced) potential of the system.

Figure 1.01 also shows that at very small distances (corresponding to high energy interactions), the strength of the force (which is proportional to the parameter $\alpha_s$) becomes very small. This allows perturbative calculations to be performed on the hard scattering interactions (giving rise to the scattering amplitudes) which can be used to evaluate cross sections. The hard scattering takes place on very short time scales so that the partons (quarks and gluons) can be considered to be free particles. This concept is known as asymptotic freedom. At large distances, Figure 1.01 also shows that the strength of the force becomes very large. This concept is known as confinement (the quarks are confined into the hadrons and cannot ever be free) or ‘infra-red slavery’[1].

![Figure 1.01: Variation of the strength of the strong interaction with distance.](image)
1.1.3 Perturbative QCD Calculations

Theoretical calculations can be performed using perturbative QCD leading to predictions of observables. These calculations use methods involving Feynman diagrams derived from the QCD Lagrangian density given in Equation 1.02. Such diagrams involve quark radiation and gluon splitting vertices such as those illustrated in Figure 1.02.

![Feynman diagrams](image)

**Figure 1.02:** Quark radiation and gluon splitting.

However, Feynman diagrams can also contain loops, as illustrated in Figure 1.03. These produce ultraviolet (UV) divergences in the calculation, which arise from the integration over the unconstrained loop momenta. These divergences are removed by a procedure called *renormalization*, where they are swept into terms that represent non-observable quantities. However, this renormalization procedure can only be done by introducing an additional momentum scale in the form of an unphysical parameter known as the *renormalization scale*, denoted by $\mu_R$. This has the consequence that the strong coupling constant, $\alpha_s$, becomes dependent on this renormalization constant. This parameter can be regarded as the momentum scale at which subtractions are done to remove UV divergent terms.
Since $\mu_R$ is an unphysical and arbitrarily chosen parameter, the value of any observable, $O$, if calculated to all possible orders (i.e., using all possible Feynman diagrams), should be independent of $\mu_R$. This requirement is expressed mathematically by the Renormalization Group Equation:

$$\frac{dO}{d\mu_R^2} = \left( \mu_R^2 \frac{\partial}{\partial \mu_R^2} + \mu_R^2 \frac{\partial \alpha_s}{\partial \mu_R^2} \frac{\partial}{\partial \alpha_s} \right) O = 0. \quad (1.07)$$

In other words, in order to keep the observable, $O$, independent of $\mu_R$, any change in $\mu_R$ must be compensated by a change in the renormalized coupling constant, $\alpha$. The renormalization scale dependence of the strong coupling constant is determined through the QCD Callan-Symanzik $\beta$-function given by[2]:

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -\sum_{i \geq 0} \beta_i (\alpha_s)^i = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \beta_2 \alpha_s^4 - \beta_3 \alpha_s^5 - \ldots, \quad (1.08)$$

where the first few coefficients $\beta_i$ are given by [3,4,5,6]:

$$\beta_0 = \frac{33 - 2n_f}{12} \quad \beta_1 = \frac{1}{16} \left( 102 - \frac{38}{3} n_f \right) \quad (1.09)$$

$$\beta_2 = \frac{1}{64} \left( \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2 \right)$$

Figure 1.03: Feynman loop diagram that leads to a UV divergence.
and $n_f$ is the number of quarks with rest mass smaller than the energy scale.

There are several schemes where the $\beta$-function is mass independent. The scheme relevant to this thesis is the modified MS scheme, denoted $\overline{MS}$ [7]. For this class of schemes, the values of $\beta_0$ and $\beta_1$ are universal[4], but the values of $\beta_2$, given in Equations 1.09 are specific to the $\overline{MS}$ scheme[8]. The values of the $\beta_i$ in the $\overline{MS}$ scheme are shown in Table 1.02.

<table>
<thead>
<tr>
<th>$n_f$</th>
<th>$\beta_0^{(n_f)}$</th>
<th>$\beta_1^{(n_f)}$</th>
<th>$\beta_2^{(n_f)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$\frac{9}{4}$</td>
<td>4</td>
<td>$\frac{3863}{384}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{25}{12}$</td>
<td>$\frac{77}{24}$</td>
<td>$\frac{21943}{3456}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{23}{12}$</td>
<td>$\frac{29}{12}$</td>
<td>$\frac{9769}{3456}$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{7}{4}$</td>
<td>$\frac{13}{8}$</td>
<td>$\frac{65}{128}$</td>
</tr>
</tbody>
</table>

**Table 1.02**: Table showing values of $\beta_i^{(n_f)}$ in the $\overline{MS}$ scheme for various orders, $i$, and number of flavours, $n_f$.

Solving Equation 1.08 for $\alpha_s$ introduces an arbitrary constant of integration. There is no theoretical means of determining the value of this constant and so it has to be determined from experiment. This is equivalent to choosing a reference scale (arbitrary value of $\mu_R$) and through experiment measuring a value of $\alpha_s$ at this value. $\alpha_s$ for all other scales can be determined from Equation 1.07. By convention, the reference scale is taken to be the
mass of the $Z^0$ boson, i.e., $\mu_R = M_Z = 91.1882 \pm 0.0022$ GeV[9]. This value is known to relatively high precision in comparison to $\alpha_s(M_Z)$.

Another approach to the arbitrary constant, is to introduce a dimensionless parameter, $\Lambda_{QCD}$ (often shortened to $\Lambda$), known as the fundamental parameter of QCD. This parameter gives the value at which, $\alpha_s$ becomes too large for the perturbative approach to be valid. It is defined by the equation[2]:

$$\ln\left(\frac{\mu_R}{\Lambda_{QCD}}\right) = -\frac{1}{2} \int_{\alpha_s(\mu_R)}^\infty \frac{dx}{\beta(x)} = \int_{\alpha_s(\mu_R)}^\infty \frac{dx}{2\beta_0 x^2 (1 + b_1 x + \ldots)}.$$  \hspace{1cm} (1.10)

It is now possible to solve the Renormalization Group Equation 1.07 in terms of this new parameter. To leading order (LO), i.e., up to the $\beta_0$ term in Equation 1.08, the integral solution becomes[2]:

$$\alpha_s(\mu_R) = \frac{1}{2\beta_0 \ln\left(\frac{\mu_R}{\Lambda_{QCD}}\right)} = \frac{12}{2(33 - 2n_f)\ln\left(\frac{\mu_R}{\Lambda_{QCD}}\right)}.$$  \hspace{1cm} (1.11)

As before, the value of $\Lambda_{QCD}$ depends on the renormalization scheme at higher orders. It also depends on the number of active flavours that are present at the scales under study (denoted $n_f$). At higher energy scales the mass of the heavy quarks at or below that scale become important. $\Lambda_{QCD}$ for different numbers of active flavours must satisfy the continuity condition for $\alpha_s$ at the energy scale boundary represented by the mass of the next quark (e.g., the boundary between $n_f = 4$ and $n_f = 5$ occurs at the mass of the $b$-quark).
1.2 Hadron-Lepton Interactions

Perturbative QCD can be tested by probing a hadron with a point-like probe such as a lepton. If the virtual boson probe (most commonly a photon) that is exchanged during the collision between the hadron and lepton (known as an event) has sufficiently high momentum, then it can have enough resolution to be able to probe the individual constituents of the hadron (i.e., the partons\(^3\)). The resolving power of the virtual boson that is exchanged is related to its four-momentum by the variable \(q^2\) (the photon four-momentum squared). Since the photon is virtual, \(q^2\) is negative and we use the variable, \(Q^2 = -q^2\). The distance that can be resolved by the photon is given by \(\lambda \propto \frac{1}{\sqrt{Q^2}}\). The parton in the hadron that interacts with the photon is called the struck parton and can either be a quark or (indirectly) a gluon. Due to asymptotic freedom, this parton is quasifree inside the hadron, if \(\alpha_s\) is small. So perturbative QCD calculations are applicable. This type of hadron-lepton interaction is known as Deep Inelastic Scattering (DIS) and is described in Chapter 2. At the HERA collider, positrons (or electrons) are collided with protons in order to test the pQCD calculations.

Figure 1.04 represents a schematic method for breaking down the processes undergone during a electron-proton (ep) collision at HERA. The collision is broken down into three main parts, labelled A, B and C.

\(^3\) The term ‘parton’ is a generic term for quark, antiquark or gluon.
(A) QED (or electroweak) radiation: the electron interacts with the parton in the proton via the emission of a boson. The four-momentum of this boson is denoted as \( q \). This is a QED process and is well understood.

(B) Hard scatter: the core interaction between the boson emitted from the lepton and the struck parton from the hadron. It represents the short distance (high \( Q^2 \)) part of the interaction, where asymptotic freedom applies and \( \alpha_s \) is small. This process can be calculated using pQCD.

(C) Proton Parton Density Function (PDF): This gives the probability of striking a quark carrying a fraction of the proton momentum, \( x \), for a given value of \( Q^2 \). It represents the complicated structure of the proton and cannot be calculated analytically. It is obtained using various fitting techniques using data from this and other experiments. To distinguish between processes that are happening inside the hadron and those that should be included in the hard
scatter, a factorization scale is introduced, denoted $\mu_F$. Once the PDF has been obtained for a specific factorization scale, it is completely independent of the hard scatter, i.e., the PDF and the hard scatter process are completely factorizable.

To calculate cross sections for an $ep$ interaction, the calculation involves a convolution of the three parts described above:

$$\text{cross section} = \text{QED radiation } \otimes \text{hard scatter } \otimes \text{hadronic PDF}. \quad (1.12)$$

Conversely, if the cross section is known from experiment and two of three components listed above are also known, then the third can be obtained. This is described in more detail in Section 2.3.2.

The cross section depends on the strong coupling constant, $\alpha_s$, the renormalization scale factor, $\mu_R$ and the factorization scale, $\mu_F$. The result of the hard scatter is a number of colour charged partons that cannot be detected directly. They undergo a process of conversion into colourless hadrons, which form the hadronic final state. These hadrons emerge from the interaction point in a highly collimated beam called a jet, following the direction of the seeding parton, before being seen in the detector. By measuring the cross section of the resultant jets, we can extract information about the hard scattering process and this can be compared to calculations done using pQCD. Thus the value of the strong coupling constant can be established. This value has been measured a number of times before, using a variety of techniques. The world average is $\alpha_s(M_Z) = 0.1182\pm0.0027[10]$. The results obtained from the analyses in this thesis include more precise measurements.
of $\alpha_s(M_Z)$. These jet cross sections can also be used to improve the fits used in producing the PDFs, which is a topic for another thesis.

1.2.1 Jet Cross Section Calculations

The general equation for calculating a differential jet cross section from ep collisions (i.e., for the process $\sigma(ep \rightarrow e + X + \text{jet})$) is given by the equation

\[
\frac{d\sigma_{\text{jet}}}{dx} = \sum_{j=q,q,g} \int dx \cdot f_{\gamma/e} \cdot f_j(x, \mu_F) \cdot d\hat{\sigma}(x, \alpha_s(\mu_R), \mu_R, \mu_F),
\]

(1.13)

where $f_{\gamma/e}$ represents the electroweak radiation from the electron (A), $f_j$ represents the proton PDF (C) and $d\hat{\sigma}$ represents the partonic cross section calculated from the hard scatter (B) (see Figure 1.04). The sum is over the (anti)quark and gluon PDFs. $d\hat{\sigma}$ is calculated from the appropriate Feynman diagrams, up to the appropriate order using Feynman rules derived from quantum field theory. This is the heart of the perturbative QCD calculation. The sum is over the available types of struck parton in the proton.

![Figure 1.05](image.png)

**Figure 1.05:** QCD DIS processes up to $O(\alpha_s)$ in the laboratory frame involved in jet production: (a) Born process (b) Boson Gluon Fusion (BGF) (c) QCD Compton (QCDC).
Figure 1.06: Feynman diagrams showing loop corrections to Born diagram.

Figure 1.07: Feynman diagrams showing example loop corrections to BGF diagrams.

Figure 1.08: Feynman diagrams showing example loop corrections to QCDC diagrams.
The first analysis in this thesis measures inclusive jet cross sections (i.e., all jets in an event). The Leading Order (LO) diagram is given by the Born process shown in Figure 1.05(a). These processes are not sensitive to the strong coupling constant, $\alpha_s$ and are largely removed by the cut on transverse energy in the Breit frame as described in the Section 3.2.1. The NLO processes Boson-Gluon Fusion (BGF) and QCD Compton (QCDC) scattering are shown in Figure 1.05(b),(c). These processes have two partons in the final state and depend directly on $\alpha_s$. This arises from the quark-gluon vertex in each diagram (also see Figure 1.02). Next-to-Leading Order (NLO) corrections include the virtual gluon loop diagrams shown in Figure 1.06. These show the NLO corrections to the LO Born process that must be calculated.

The second analysis in this thesis uses exclusive dijet events (i.e., only events with exactly two jets produced). Figure 1.05(b),(c) also represents the leading order process for this type of event. Figures 1.07 and 1.08 show some of the virtual loop correction diagrams that are required to calculate the NLO predictions. Figure 1.09 shows some of the higher order diagrams also required. These diagrams produce trijet events and are the LO diagrams for such trijet event calculations. They are needed in the calculation of the
exclusive dijet events in order to cancel some of the singularities that arise from the virtual loop correction diagrams, i.e., both real and virtual corrections are needed to keep the calculations finite. Perturbative QCD calculations for the jet cross sections in the analyses in this thesis are performed up to Next-to-Leading Order[1].

1.3 Thesis Goals

This thesis describes two analyses that aim to measure the value of $\alpha_s$ to a greater accuracy than has presently been done to date. In order to do this, measurements of both inclusive jet cross sections and exclusive dijet cross sections have been made in deep inelastic scattering electron-proton interactions from a data sample collected by the ZEUS detector at the HERA accelerator in Hamburg. This is the largest data sample so far used for such analyses at HERA. These measurements are then compared to predictions made by NLO computation programs using perturbative QCD, in order to test the accuracy of these predictions and hence the validity of the QCD theory used to make them. This comparison between data measurements and the NLO predictions is then used to extract precise values of the strong coupling constant, $\alpha_s$, as well as to test its dependence on scale [11].

The importance of this parameter is twofold. Firstly it is the parameter of pQCD. It cannot yet be predicted by theory and hence has to be measured. Consequently, there is no limit to the precision with which we would like to measure it. Should a theory eventually arise that predicts the value of $\alpha_s$, its validity will be judged on the comparison of that theory with the most precise measurements of the time. Secondly the value of this
parameter is required as an input to current theories; not only QCD, which is part of the Standard Model, but also more exotic theories such as Supersymmetry, String Theory and Grand Unified Theories.

This thesis is organized as follows. Chapter 1 provides an overview of the Standard Model and the theoretical framework underlying QCD and hadron-lepton interactions. Chapter 2 deals with some further theoretical considerations concerning the types of events recorded, including deep inelastic scattering and structure functions. Chapter 3 describes the jet algorithm used and the Breit frame. Chapter 4 describes the HERA experiment and the ZEUS detector in detail, including how the data for the analyses were obtained. Chapter 5 discusses Monte Carlo reconstructed events. This is used to produce simulated data that are used to make corrections on the actual ZEUS data used for these analyses. Chapter 6 outlines in more detail how the NLO calculations are performed. These are used to make theoretical predictions from which values of $\alpha_s$ are extracted. Chapter 7 describes the data sample in more detail, including how events were selected. Chapter 8 discusses the correction factors that are used on the data and how Monte Carlo samples are used to obtain them. In Chapter 9 results are presented in the form of jet cross section plots. Comparisons between the data measurements and the theoretical predictions are shown including the relevant uncertainties. In Chapter 10 the results are analyzed and values of $\alpha_s(M_Z)$ are extracted with the corresponding uncertainties. Finally, in Chapter 11, overall conclusions are presented with an outlook to future work. A brief presentation of an ongoing analysis using jets that have been tagged with a charm quark is included.
2 Event Processes

2.1 Deep Inelastic Scattering

Deep Inelastic Scattering (DIS) is the scattering of a lepton (usually an electron or positron) off a hadron (e.g., protons or even whole nuclei), where the momentum exchange during the process is large. It has proved to be a very useful technique to test the Standard Model. It happens via the exchange of a virtual boson. This may be either a virtual photon, $\gamma$, or a weak vector boson, $Z^0$, in which case the process is called Neutral Current (NC) DIS; or one of the charged weak vector bosons, $W^\pm$, in which case the process is called Charged Current (CC) DIS. This latter process causes the electron or positron to change into a neutrino and so is a lepton flavour changing process. The cross sections for NC and CC processes are roughly equal for high boson virtualities (denoted by $Q^2$), i.e., for $Q^2 > 10000 \text{ GeV}^2$ (approximately equivalent to the masses of the weak bosons, W and Z, squared), however CC processes are significantly suppressed for $Q^2 < 10000 \text{ GeV}^2$ in comparison to NC processes as the massless photon dominates in this region. Figure 2.01 illustrates a NC DIS event.

![Figure 2.01: Diagram showing a NC DIS event.](image-url)
The result of the DIS process is that the hadron is destroyed as a single unit, yielding a complicated multi-particle final state that can be used to deduce information about the initial hadron.

The general reaction can be represented by the following equation:

\[ e(k) + P(p) \rightarrow e'(k') + X(p'), \]

where the initial state particles \( e \) and \( P \) are defined by the experiment and the final state particles are \( e' \) (the scattered lepton) and \( X \), which represents the multi-particle state resulting from the DIS process. At HERA, electrons or positrons are collided with protons. For NC DIS processes, this is often expressed in the following way: \( e\pm p \rightarrow e\pm X \).

### 2.2 Deep Inelastic Scattering Kinematics

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre-of-mass energy</td>
<td>( s = (k + p)^2 = 4E_eE_p )</td>
<td>Fixed by the initial experimental conditions.</td>
</tr>
<tr>
<td>Virtuality</td>
<td>( Q^2 = -q^2 = -(k - k')^2 )</td>
<td>Resolving power of the exchange boson.</td>
</tr>
<tr>
<td>Bjorken ( x )</td>
<td>( x = \frac{Q^2}{2p \cdot q} )</td>
<td>Fraction of proton momentum carried by struck quark.</td>
</tr>
<tr>
<td>Inelasticity</td>
<td>( y = \frac{p \cdot q}{p \cdot k} = \frac{Q^2}{s x} )</td>
<td>Amount of energy transferred to the lepton from the proton in the proton’s rest frame.</td>
</tr>
</tbody>
</table>

*Table 2.01: Table showing DIS kinematic variables.*
For the NC DIS process shown in Figure 2.01, Table 2.01 shows the kinematic variables that are used to describe the event. The photon virtuality, $Q^2$, is also related to its wavelength by $\lambda \sim 1/\sqrt{Q^2}$, so as $Q^2$ increases the proton is probed at smaller and smaller scales. It can be seen from Table 2.01, that only two event variables from $Q^2$, $x$ and $y$ are needed to completely describe the event as they are simply related as follows:

$$Q^2 = sxy. \quad (2.01)$$

### 2.3 Structure Functions

The cross section, $d\sigma$, for deep inelastic $ep$ scattering can be factorized into a leptonic tensor, $L_{\mu\nu}$, and a hadronic tensor, $W^{\mu\nu}$. In the single boson exchange model (see Figure 2.01):

$$d\sigma = L_{\mu\nu}W^{\mu\nu}. \quad (2.02)$$

The leptonic tensor can be calculated exactly from QED and is symmetric with respect to the exchange of $\mu$ and $\nu$. The hadronic tensor is used to parameterize the proton structure and the details of the hadronic vertex. It can be written in terms of QCD proton structure functions\[1\]. The DIS cross section for $ep$ collisions can be written in terms of three out of the four structure functions $F_1, F_2, F_3$ and $F_L$:

$$\frac{d^2\sigma(e^+p)}{dx dQ^2} = \frac{4\pi\alpha_s^2}{x Q^4} \left[ Y_x F_2(x, Q^2) - y^2 F_L(x, Q^2) + y x F_3(x, Q^2) \right], \quad (2.03)$$

where $Y_\pm = 1 \pm (1 - y^2)$. $F_2$ is the contribution to the DIS cross section due to the interaction between the quarks and the transversely polarized exchange virtual bosons. It is directly related to the density of (anti)quarks in the proton. $F_L$ is known as the
longitudinal structure function and is defined as $F_L = F_2 - 2xF_1$. It is related to the absorption cross section of longitudinally polarized virtual photons. $F_3$ is the contribution to the cross section due to the parity violating $Z^0$ boson. This contribution is only significant at $Q^2 > M_Z^2$.

2.3.1 Quark Parton Model and Bjorken Scaling

The Quark Parton Model assumes that the proton is made of ‘quasi-free’ point-like partons (quarks). If this model were a true representation of the proton then probing the proton with higher and higher $Q^2$ photons should reveal no further structure as its constituents would be point-like. This would imply that the structure functions should be independent of $Q^2$, which represents the resolution of the probing. The photon should only interact with a quark carrying a momentum fraction $x$ of the proton’s total momentum [12]. In this case the structure function, $F_i$ is said to ‘scale’ with $Q^2$ and this is the so-called ‘Bjorken scaling’:

$$F_i(x, Q) \rightarrow F_i(x). \quad (2.04)$$

The quarks in the QPM are spin-$\frac{1}{2}$ fermions which can only couple to transversely polarized bosons, so as the absorption cross section of longitudinally polarized virtual photons is zero then $F_L$ vanishes[1]:

$$F_2(x) = 2xF_1(x) \quad (2.05)$$

This model has been successful at low $Q^2$ where the gluons in the proton cannot be resolved and their effects can be neglected. However, in kinematic regions where the gluons become important then the QPM fails and the structure functions become no
longer independent of $Q^2$. This is the so-called ‘scaling violation’. Figure 2.02 shows the scaling violations in $F_2$ in measurements made at HERA and other fixed target experiments[13]. Note that the HERA data span a wide kinematic region; $6 \times 10^{-5} < x < 0.65$ and $1 < Q^2 < 10^5 \text{ GeV}^2$. It can be seen that in those regions of very low $x$, the lines are not flat, i.e., $F_2$ has a strong dependence on $Q^2$. If one only considers virtual photon exchange, the structure function $F_2$ is the sum of the momentum distributions of the quarks weighted by the square of their charges:

$$F_2(x, Q^2) = \sum_i e_i^2 q_i(x, Q^2),$$

(2.06)

where the sum runs over each quark flavour, $i$, charge $e_i$ and probability density $q_i(x, Q^2)$. This scaling indicates that the proton has increased structure as it is probed at higher resolutions (at higher $Q^2$). A quark can radiate a gluon. A gluon can split into a gluon pair or a quark-antiquark pair. The closer one looks, the more detail one can see. At lower values of $x$, there is more momentum room for gluons to be radiated hence the scaling violations are seen in this region. This is illustrated in Figure 2.03.

### 2.3.2 Proton Parton Density Functions

In trying to use Equation 1.13 to calculate jet cross sections, information is needed on the struck parton from the proton. It is this parton that takes part in the QCD hard scatter process that leads to the measured jets. This information comes in the form of the proton Parton Density Function (PDF) which gives the probability of the struck parton having a momentum fraction, $x$, when the proton is being probed at a particular scale, $Q^2$. 

41
The exact nature of the dependence of the PDF on $x$ and $Q^2$ is not computable using perturbation theory, but has to be determined empirically from data from a wide range of processes (including DIS, Drell-Yan (where two high-$p_T$ leptons result from the event), jet production and prompt photons (a high-$p_T$ photon pair produced in the event)). The method usually consists of parameterizing the parton densities at some starting scale $Q_0$. 

Figure 2.02: Measurements of the structure function, $F_2$ made at HERA and other fixed target experiments.
and adjusting the parameters to fit the data. These parameters are usually constrained by two conditions:

(i) momentum sum rule: \[ \sum_a \int_0^1 dx \cdot x \cdot f_a(x, Q^2) = 1, \] (2.07)

(ii) the parton density must be positive: \[ f_a(x, Q^2) > 0. \] (2.08)

The usefulness of equation 1.13 for calculating jet cross sections relies on the universality of the proton PDFs. Once determined they can be used in the calculation of any hard scattering process involving the proton. This relies on the fact that the PDF is independent of scattering process (i.e., \( f_a(x, Q^2) \) can be separated from the rest of equation 1.13). This concept is known as factorization. The factorized PDF can be written in the form:

![Figure 2.03: Variation in proton structure at decreasing values of Bjorken x.](image)

\[ P \]  
\[ q \]  
\[ quark \]  
\[ jet \]  
\[ proton \]  
\[ remnant \]  
\[ e \]  
\[ x \]  
\[ P \]  
\[ q \]  
\[ quark \]  
\[ jet \]  
\[ proton \]  
\[ remnant \]  
\[ e \]  
\[ x \]
\[
f(x, Q^2) = f(x) + \frac{\alpha_s}{2\pi} \int_x^1 dy \frac{dy}{y} q(y) f(P^0_{jk}(\alpha_s, x/y), Q), \tag{2.09}
\]

where the kernels, \( P^0_{jk}(\alpha_s, z) \) are the so-called Altarelli-Parisi splitting functions, which give the probability that a parton, \( k \), emitting a parton, \( j \), carried a momentum fraction, \( z \), of the original parton. The four leading order splitting functions are shown in Figure 2.04 [1,2,14].

These correspond to a quark radiating a gluon, \( q \to qg \), a quark containing a gluon, \( q \to gq \), a gluon splitting into a quark-antiquark pair, \( g \to q\bar{q} \) and gluon splitting, \( g \to gg \). To leading order, they are given by the following equations respectively:

\[
P_{qg}^0(z) = \frac{4}{3} \left[ \frac{1+z^2}{1-z} \right] \quad P_{gq}^0(z) = \frac{1}{2} \left[ z^2 + (1-z)^2 \right] \\
P_{gq}^0(z) = \frac{4}{3} \left[ \frac{1+(1-z)^2}{1-z} \right] \quad P_{gg}^0(z) = 6 \left[ \frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right]. \tag{2.10}
\]

Although the absolute scale dependence of the proton PDF cannot be determined theoretically, Equation 2.09 can be differentiated and separated into pieces associated
with quark and gluon radiations, resulting in the following coupled equations, the
DGLAP\(^4\) evolution equations\cite{15}:

\[
\frac{dq_i(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{y} \sum q_i(y, Q^2) P_{qq} \left( \frac{x}{y} \right) + g(y, Q^2) P_{qg} \left( \frac{x}{y} \right),
\] (2.11)

\[
\frac{dg(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{y} \sum g(y, Q^2) P_{gg} \left( \frac{x}{y} \right) + q_i(y, Q^2) P_{qg} \left( \frac{x}{y} \right),
\] (2.12)

where the \( P_{jk}(z) \) represent the full splitting function with its perturbative expansion in \( \alpha_s \) included.

Once the parton density has been parameterized and determined at the input scale \( Q_0 \),
then the DGLAP Equations 2.11 and 2.12 can be evolved in \( Q^2 \) to determine the parton
density at any other scale. This is then compared to experimental data sets and the
parameterizations are adjusted iteratively until the best fit with the data sets is
achieved\cite{16}. Example PDFs are shown in Figure 2.05, including the MRST,
ZEUS-S\cite{17} and CTEQ6M\cite{18} PDFs.

2.3.3 Modern PDF Parameterizations

2.3.3.1 Martin-Roberts-Stirling-Thorne (MRST)

The MRST PDF parameterization uses NLO DGLAP evolution. It uses the following
functional form for the quark and gluon distributions with a starting scale of \( Q_0^2 \) in the
\( \overline{MS} \) renormalization scheme:

\[
xf_i(x, Q_0^2) = A_i \cdot x^\delta \left( 1 - x \right)^\eta \left( 1 + \epsilon, \sqrt{x} + \gamma_i x \right),
\] (2.13)

\(^4\) DGLAP: Dokshitzer-Gribov-Lipatov-Altarelli-Paresi
where \( f_i \) is a certain parton density and \( \{ A_i, \delta, \eta, \gamma \} \) are the parameters to be fitted. Some of the \( A_i \) are not free but instead fixed by flavour or momentum sum rules (Equation 2.08). The density of charm quarks is assumed to be zero at the starting scale, \( Q_0^2 \) and is generated at higher scales through gluon splitting (Figure 2.04) using the DGLAP equations. The MRST99[19] parameterization published by this group is the main one used for the analyses in this thesis.

### 2.3.3.2 Coordinated Theoretical Experimental Project on QCD (CTEQ)

This group uses a similar method to the MRST group. For the quark distribution the same functional form as the MRST group is used (Equation 2.13), however, for the gluon distribution, the following functional form for the parameterization is used:

\[
x_g(x, Q_0^2) = A_0 x^A (1 - x)^A (1 + A_1 x x^A).
\]

(2.14)

![Figure 2.05: Proton PDFs (multiplied by \( x \)) for the up/down valence quarks, the sea quarks and the gluons (denoted by \( u_v, d_v, s \) and \( g \) respectively).](image)
Two further differences between the CTEQ and MRST methods are:

(i) CTEQ uses different data sets to constrain the strange quark distribution.

(ii) For CTEQ, the charm quark distribution is not constrained below the mass threshold but instead uses the ‘variable flavour technique’.

As a systematic check the PDF parameterization provided by CTEQ4[20] is used in the analyses in this thesis. Like the MRST99 PDFs, they have been done using multiple values of $\alpha_s(M_Z)$. 
3 Jet Physics

Due to the confinement of quarks and gluons within colourless hadrons, it is impossible to directly detect the struck quark or gluon from the DIS process. Through a process of parton showering and hadronization, the struck parton (and any other partons that result from the immediate hard scatter) becomes a highly collimated stream of hadrons known as a jet, which is detected as tracks in the tracking detector and localized groups of energy deposits in the calorimeter. From this detector information, these jets are reconstructed in terms of their four-momenta, which in turn, through momentum and energy conservation, provide information about the partons that seeded them. In short, by reconstructing jets in the detector, a direct handle on the underlying QCD processes can be obtained. However, for this to work meaningfully, jets must be reconstructed in such a way that they can be mapped directly back to the seeding partons from the hard scattering process, which requires the implementation of a good jet algorithm.

3.1 Jet Algorithms

A jet algorithm is a software program which allows jets to be reconstructed from a final state set of hadrons or a set of energy deposits/tracks seen in a detector. For a jet algorithm to be useful, it requires certain features:

(i) It must be collinear and infra-red safe[21]. This means that jet cross sections are insensitive to divergences arising from particles in the jets moving in parallel.
There must be good correspondence between the jets found at the detector level with jets found at the hadron level and the corresponding seed parton. In other words, corrections for the hadronization that occurs between the parton and hadron levels should be small.

It must be longitudinally invariant, in that the quantities used to describe the jet must be invariant under longitudinal boosts (i.e., boosts along the direction of the initial hadron).

The reconstructed jets must not be sensitive to soft processes in which a gluon is radiated by a quark in the jet with low transverse energy relative to the jet.

Commonly used jet algorithms generally fall into two categories; cone algorithms and cluster algorithms. The jet algorithm used here is a cluster algorithm.

### 3.1.1 Cluster Algorithms

There are several versions of the cluster algorithm available\cite{22,23} but the most commonly used one that is relevant to this thesis is the \textit{longitudinally invariant k}_T-\textit{cluster algorithm}\cite{24}. This algorithm uses the following method\cite{25}:

(i) The particles (hadrons, partons or energy deposits) are described by the following variables; transverse energy, \(E_T\), azimuthal angle, \(\phi\), and pseudorapidity, \(\eta\), which are defined by the following equations:

\[
E_T = E \sin \theta \quad \quad \eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right].
\] (3.01)
where \( \theta \) is the polar angle, i.e., the angle between the hadron beam momentum vector (which defines the \( z \)-axis) and the particle momentum vector. Pseudorapidity is a useful quantity because it is the longitudinally invariant polar angle. It is useful as Lorentz boosts along the \( z \)-axis change this quantity only by an additive constant, which means that differences in pseudorapidity are invariant under such Lorentz transformations.

(ii) For each pair of particles, \( i \) and \( j \), a distance parameter, \( d_{ij} \), is defined by the following equation:

\[
d_{ij} = \min\{E_{T,i}, E_{T,j}\} \left[ (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 \right].
\]  

(iii) For each single particle, \( i \), a distance parameter, \( d_i \), from the particle to the beam is defined by the following equation:

\[
d_i = E_{T,i}.
\]  

(iv) The minimum of all the \( \{d_{ij}, d_i\} \) is taken. If this is a \( d_i \), then the corresponding particle is considered to be a protojet and removed from the sample. However if this minimum is a \( d_{ij} \), then the two corresponding particles are combined to a single particle, \( k \), according the following equations (the Snowmass convention)[26]:

\[
E_{T,k} = E_{T,i} + E_{T,j} \quad \eta_k = \frac{1}{E_{T,k}} \left( \eta_i E_{T,i} + \eta_j E_{T,j} \right) \quad \phi_k = \frac{1}{E_{T,k}} \left( \phi_i E_{T,i} + \phi_j E_{T,j} \right)
\]  

(v) The above process is repeated until all the particles have been assigned to jets. After each iteration of the algorithm, the number of remaining particles in the
sample is reduced by one, either by direct removal as a protojet or combining two particles into one according to Equations 3.04.

(vi) A set of cuts on $E_T$ and $\eta$ can be made to get the final sample of jets.

There are a number of advantages to this cluster algorithm over the cone algorithm[24,27,28].

(i) The cluster algorithm does not impose any geometry on jet shape. However, a radius parameter can be introduced:

$$d_i = E_{T,i}^2 \cdot R \tag{3.05}$$

which effectively defines the maximum size of a jet in the $\eta$-$\phi$ plane. Generally R is set to one.

(ii) The final jet sample is less influenced by soft particles close to the jet. This leads to smaller detector and hadronic corrections.

(iii) Jets are completely separated from the proton remnant. The proton remnant generally has very small transverse energy, so very forward jets close to the beam-pipe can be reconstructed.

(iv) Unlike the cone algorithm, there is no issue with overlapping jets with this cluster algorithm.

(v) The $k_T$-cluster algorithm is considered to be completely infra-red safe to all orders of QCD.
3.2 The Breit Frame

In the Breit reference frame[29], commonly used at HERA for jet analyses in NC DIS, the struck quark (or parton) from the proton strikes the exchange boson head-on. The axis of the collision defines the $z$-axis. In the QPM, the struck quark exits the collision with equal and opposite momentum, hence this frame is also known as the ‘Brick Wall Frame’ (see Figure 3.01). There is no energy transfer between the leptonic and hadronic sides of the collision process. The following equation is then satisfied and defines the Breit Frame:

$$2x\vec{p} + \vec{q} = 0,$$  \hspace{1cm} (3.06)

where $x$ is the fraction of the proton’s total momentum carried by the struck parton and $\vec{p}$, $\vec{q}$ are the momenta of the proton and exchanged boson respectively. In the Breit frame, the four-momentum of the exchanged boson is purely space-like. The four-momenta of the participating particles in the QPM are:

$$\vec{q} = (0,0,0,-Q) \hspace{0.5cm} \vec{p}_q = \left(\frac{Q}{2},0,0,\frac{Q}{2}\right) \hspace{0.5cm} \vec{p}_q' = \left(\frac{Q}{2},0,0,-\frac{Q}{2}\right)$$  \hspace{1cm} (3.07)

where $\vec{p}_q$ is the initial momentum of the struck parton and $\vec{p}_q'$ is the final momentum of the same parton.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{breit_frame_diagram}
\caption{Born process in the Breit Frame.}
\end{figure}
At HERA, physics events are reconstructed in the laboratory frame, so for jets to be reconstructed in the Breit frame all quantities need to be boosted into that frame. In the laboratory frame the exchange boson four-momentum, \( q = (q_0, q_1, q_2, q_3) \), can be written in terms of event variables according to the following equations:

\[
q_0 = y \cdot (E_e - xE_p) \quad q_1 = \sqrt{Q^2 (1-y)} \quad q_2 = 0 \quad q_3 = -y \cdot (E_e + E_p).
\]  

(3.08)

The second component is arbitrarily set to zero. A rotation about the \( z \)-axis does not change the frame from being a ‘Breit Frame’, so this is the convention that is followed. A vector, \( \vec{p}_L \), in the laboratory frame can be boosted into a vector, \( \vec{p}_B \), in the Breit frame using a Lorentz Boost Matrix, \( L_{LB} \):

\[
\vec{p}_B = L_{LB} \cdot \vec{p}_L.
\]

(3.09)

The return boost uses the inverse Lorentz Boost Matrix, \( L_{BL} = (L_{LB})^{-1} \):

\[
\vec{p}_L = L_{BL} \cdot \vec{p}_B.
\]

(3.10)

The Lorentz Boost Matrices for boosting from the laboratory frame to the Breit frame and back are given by the 4×4 matrices:

\[
L_{LB} = \begin{pmatrix}
\frac{q_0 + Q}{Q} & -\frac{q_1}{Q} & -\frac{q_2}{Q} & -\frac{q_3}{Q} \\
\frac{q_0 - q_3}{Q} & \frac{q_1}{Q} & \frac{q_2}{Q} & \frac{q_3 - q_0}{Q} \\
\frac{q_0 - q_3}{Q} & -\frac{q_1}{Q} & -\frac{q_2}{Q} & -\frac{q_3}{Q} \\
0 & \frac{q_2}{Q} & -\frac{q_1}{Q} & 0
\end{pmatrix}
\]

(3.11)

and
3.2.1 Jet Production in the Breit Frame

As discussed in Section 1.2.1, jet production can be directly related to the underlying QCD processes that occur during the collision. Up to $O(\alpha_s)$, the Feynman diagrams in Figure 3.02 represent the possible QCD processes in the laboratory frame. Figure 3.03 shows the same processes in the Breit Frame.

In diagrams 3.02 (a) and 3.03(a), the lowest order process in jet production is shown and is known as the Born Process. Here, the struck quark absorbs the exchanged boson and then goes on to initiate a jet. In the Breit frame this process happens along the $z$-axis with all the particles having little or no transverse energy. This process is purely QED and has no dependence on $\alpha_s$ and therefore no QCD dependence. For diagrams 3.02(b),(c) and 3.03(b),(c), known respectively as Boson-Gluon Fusion (BGF) and QCD Compton (QCDC) processes, two jets are resultant, which must have equal and opposite transverse energy. These are QCD processes and are directly dependent on $\alpha_s$. In studying jets with a minimum transverse energy in the Breit frame (e.g., $E_T > 5 \text{ GeV}$), Born process events...
are suppressed and only events that depend on QCD processes are enhanced. Hence by measuring jet cross sections, there is a direct handle on the QCD process and a direct test of QCD theory can be made. Another advantage of the Breit frame with a cut on transverse energy of the jets is that there is maximal separation between the jets from the hard interaction and the proton remnant, so that the proton remnant does not pollute the jet sample. This arises because, in the Breit Frame, the struck quark travels in the negative $z$-direction and the proton remnant travels in the positive $z$-direction, whereas in the laboratory frame the proton remnant and the struck quark are usually moving in similar directions.

**Figure 3.02**: QCD DIS processes up to $O(\alpha_s)$ in the laboratory frame involved in jet production:  
(a) Born process (QPM)  (b) Boson Gluon Fusion (BGF)  (c) QCD Compton (QCDC).

**Figure 3.03**: QCD DIS processes up $O(\alpha_s)$ in the Breit frame involved in jet production:  
(a) Born process (QPM)  (b) Boson Gluon Fusion (BGF)  (c) QCD Compton (QCDC).
4 Experimental Setup: The ZEUS Detector at HERA

4.1 HERA

The HERA\textsuperscript{5} accelerator is located at the DESY\textsuperscript{6} Research Institute in Hamburg, Germany and is the first and only proton-electron\textsuperscript{7} collider in the world. It has been in operation since 1992 and has been called the ‘world’s most powerful electron microscope’. It was designed to allow DIS to be studied at higher centre-of-mass energies and in a wider kinematic range than the fixed targets experiments had made available at that time. The HERA tunnel is built at a depth of 10 to 25 m and has a circumference of 6.3 km. The ring is made up of four circular arcs of radius 779 m. These four arcs connect at four interaction points (IPs) at which four detector experiments are located. The layout of the ring is shown in Figure 4.01.

The HERA ring itself actually consists of two pipes, one carrying electrons clockwise (as seen from above) with an energy of 27.5 GeV and the other carry protons anticlockwise with an energy of 920 GeV\textsuperscript{8}. These two beams are brought into collision at two of the interaction points, in the north and south experimental halls, which are encompassed by two multipurpose detectors, H1 and ZEUS. In the east and west experimental halls, there are two fixed target experiments. The HERMES experiment, in the east, studies the spin structure of the nucleon using collisions of polarized electrons on a polarized gas target.

\textsuperscript{5} HERA stands for Hadron-Electron Ring Accelerator (Anlage).
\textsuperscript{6} DESY stands for Deutches Elektronen SYnchrotron.
\textsuperscript{7} As regards collisions in the HERA accelerator, positron and electron can be used interchangeably.
\textsuperscript{8} From 1992 – 1997 the proton beam energy was 820 GeV. This was increased to 920 GeV in 1998.
The HERA-B experiment, in the west, was set up to study CP violation in the B system using the collisions of the protons on a wire target.

The injection system for the two beams is enclosed in the main DESY site on the west side of the HERA tunnel. Beam injection is implemented through several pre-accelerator systems. The electron injection system uses the following method:

(i) The electrons are accelerated up to 200 MeV using the LINAC II accelerator.

(ii) They are then accelerated to 7.5 GeV using the DESY II synchrotron.

(iii) Further acceleration up to 14 GeV using the PETRA accelerator then follows.

Figure 4.01: Layout of the HERA accelerator complex.
Finally, the electrons are injected into the main HERA ring for final acceleration up to 27.5 GeV.

The proton injection system uses the following method:

(i) Hydrogen ions, $H^-$, are accelerated to 50 MeV in the LINAC.

(ii) These ions are then accelerated to 7.5 GeV in the DESY III accelerator where the electrons are then stripped off to leave the bare protons.

(iii) These protons are then accelerated to 40 GeV in the PETRA ring.

(iv) Finally the protons are injected into the main HERA ring for final acceleration up to 920 GeV.

The protons and electrons are injected into the HERA ring and travel in bunches. A maximum of 210 bunches are available for both electrons and protons, separated by 96 ns (about 28.8 m). Some of these bunches are kept empty and are known as *pilot bunches*. The purpose of these is to study background conditions in the ZEUS and H1 detectors. Where an empty bunch from one beam crosses a full bunch from the other beam there is no chance of an $ep$ collision, so any signal will most likely be the result of a collision between the beam (corresponding to the filled bunch) and the residual gas in the beam pipe. This is called ‘beam related background’. Where two empty bunches cross, non-beam related background can be estimated, most often from cosmic rays.
4.2 The ZEUS Detector

The ZEUS detector is one of the two multipurpose detectors situated around the HERA ring, specifically designed to study ep collisions. It is located about 30 m underground in the south experimental hall and is the focus of the ZEUS Collaboration, which consists of around 450 physicists from 12 countries. It is a quasi-hermitic detector covering about 99.7% of the 4π solid angle (most of the 0.3% is accounted for by the entry and exit points of the beam pipes) and has dimensions 12m × 11m × 20m. The layout of the detector is shown in Figure 4.02. A detailed description of the detector and its components can be found in [30]. However, the important components relevant to the analyses described in this thesis are outlined in the following sections.

4.2.1 Luminosity Monitor

In order to ascertain cross sections for processes measured in the detector, a precise measurement of the time-integrated luminosity is required. The ep luminosity at ZEUS is measured using the luminosity monitor[31], which measures the rate of bremsstrahlung events, ep → epγ from the Bethe-Heitler process [32], where the electron and the photon are both scattered by a very small angle. The cross section, σB, of this process at fixed photon angle θγ and energy Eγ is well known and understood to within an accuracy of 0.5%. It is given semi-classically by the Bethe-Heitler formula:

9 The ZEUS detector is described here as it was during the period 1996-2000. It is during this period that the data relevant to the analyses in thesis were taken. From 2000 to 2002, HERA and ZEUS underwent a major upgrade, as a result of which, the beam luminosities were increased by a factor of five and two new detector components were installed into ZEUS; the MicroVertex Detector (MVD) and the Straw Tube Trackers (STT), which are located around the interaction point and in the forward tracking region respectively. Data taking by ZEUS at HERA II, as it is now known, has been taking place since 2002 and due to end in mid-to-late 2007.
\[
\frac{d\sigma_{\gamma\gamma}}{dE_\gamma} = 4\alpha^2 \frac{E_e'}{E_\gamma E_e} \left( \frac{E_e'}{E_e} + \frac{E_e}{E_e'} - 3 \right) \ln \left( \frac{4E_p E_e E_e'}{MmE_\gamma} - 1 \right) - \frac{1}{2},
\]

(4.01)

where \(E_\gamma\) is the photon energy, \(E_e\) and \(E_e'\) are the initial and final electron energies, \(E_p\) is the proton energy, \(M\) is the proton mass, \(m\) is the electron mass, \(r_c\) is the classical electron radius and \(\alpha\) is the QED fine structure constant.
The luminosity of the $ep$ interactions is given by the general luminosity equation:

$$L = \frac{N_\gamma}{\sigma_B}$$  \hfill (4.02)

where $N_\gamma$ is the number of bremsstrahlung photons detected in a given time. This quantity is measured using the luminosity monitor shown in Figure 4.03. This consists of a photon (LUMI-$\gamma$) and an electron (LUMI-e) calorimeter, located next to the beam pipe at $z = -107$ and $z = -35$ m, respectively, which have resolutions given by $\frac{\sigma(E)}{E} = 0.23 \frac{\sqrt{E}}{E}$ and $\frac{\sigma(E)}{E} = 0.18 \frac{\sqrt{E}}{E}$ respectively, with $E$ measured in GeV.

The photon calorimeter is shielded by a lead-carbon filter to protect it from synchrotron radiation. The bremsstrahlung event rate, $N_\gamma$, is then determined by counting the number of photons in the photon calorimeter above a fixed threshold. To calibrate the energy scale of the photon calorimeter, coincident energy deposits in both the photon and electron calorimeters are used as the total energy of the deposits should sum to the initial electron beam energy.

![Luminosity Monitor](image)

**Figure 4.03:** ZEUS luminosity monitoring system.
The main background is due to the bremsstrahlung from the electrons in the beam-pipe gas. This is estimated using the pilot bunches (i.e., the electron bunches coincident with an empty proton bunch).

The integrated luminosities produced by HERA, recorded on ZEUS tape (after rejection of data not considered useful for physics analysis by offline data quality monitoring) and used for physics analyses during the running periods 1993–2000 are shown in Table 4.01. The average efficiency of the ZEUS detector during this time was around 70%. The HERA luminosities shown in Table 4.01 are also shown graphically in Figure 4.04.

<table>
<thead>
<tr>
<th>Running Period</th>
<th>HERA delivered/pb(^{-1})</th>
<th>ZEUS on-tape/pb(^{-1})</th>
<th>ZEUS physics data/pb(^{-1})</th>
<th>ZEUS physics events/10(^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993-4 (e(^-))</td>
<td>2.17</td>
<td>0.99</td>
<td>0.82 ± 1.5%</td>
<td>8.34</td>
</tr>
<tr>
<td>1994-5 (e(^+))</td>
<td>17.42</td>
<td>10.57</td>
<td>9.64 ± 1.2%</td>
<td>29.98</td>
</tr>
<tr>
<td>1996-7 (e(^+))</td>
<td>53.51</td>
<td>40.54</td>
<td>38.62 ± 1.6%</td>
<td>50.58</td>
</tr>
<tr>
<td>1998-9 (e(^-))</td>
<td>25.20</td>
<td>17.78</td>
<td>16.67 ± 1.8%</td>
<td>23.57</td>
</tr>
<tr>
<td>1999 – 2000 (e(^+))</td>
<td>94.95</td>
<td>73.37</td>
<td>66.04 ± 2.25%</td>
<td>66.76</td>
</tr>
</tbody>
</table>

**Table 4.01:** Table showing HERA and ZEUS integrated luminosities.

The data sets used for the analyses in this thesis were obtained in the HERA running periods between 1998 and 2000 and are shown in bold in Table 4.01.
4.2.2 ZEUS Coordinate System

The ZEUS detector has a standard cartesian coordinate system imposed on it when referring to $ep$ collisions in the laboratory frame. The direction of the proton beam through the detector defines the $z$-axis. The $y$-axis is a vertical axis through the interaction point between electrons and protons and the $x$-axis defines a radius from the interaction point at the geometric centre of the detector to the centre of the HERA ring. A spherical coordinate system is also often employed, with a polar angle given by $\theta$ and an azimuthal angle given by $\phi$. Frequently, instead of using the polar angle, $\theta$, another quantity called pseudorapidity is used, which is defined by the following equation:

Figure 4.04: HERA luminosities during running periods between 1993 and 2000.
\[ \eta = -\ln \left( \tan \frac{\theta}{2} \right) \] (4.03)

This is also known as the ‘longitudinally invariant’ polar angle. It is a useful quantity because under a Lorentz transformation along the z-axis, it only changes by an additive constant. Therefore differences in pseudorapidity are invariant under such transformations. This makes this variable useful in the definition of jet algorithms. The forward region corresponds to \( \eta > 0 \) and the backward region to \( \eta < 0 \) as shown in Figures 4.05.

### 4.2.3 ZEUS Components

The data for the analyses in this thesis were collected during the 1996–97\(^{10} \) and 1998–2000 running periods. During this time the ZEUS detector consisted of an inner set of tracking detectors that surround the central interaction point. These are divided into three sections: the forward (FTD/FDET), central (CTD) and rear (RTD) tracking detectors. These chambers are surrounded by an outer high resolution Uranium-Scintillator Calorimeter (UCAL). The UCAL is separated into three sections; the Forward (FCAL), Barrel (BCAL) and rear (RCAL) calorimeters. Behind the RTD, covering the part of the RCAL surrounding the beam pipe to a radius of about 34 cm, is the Small-angle Rear Tracking Detector (SRTD). At a depth of three radiation lengths into the RCAL, the Hadron-Electron Separator (HES) can be found. It is made up of \( 3 \times 3 \) cm\(^2 \) silicon diodes and provides an improved method of discriminating between showers due to

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\(^{10}\) For the main analyses in this thesis data sets taken obtained during the years 1998 – 2000 were used. However, the 1996 – 1997 data sets were also used in an additional analysis that is described in Chapter 11.
electromagnetic and hadronic material coming from low energy (less than 5 GeV) particles.

In the central portion of the detector, between the BCAL and the CTD lies a superconducting coil. It has an overall length of 2.46 m with an inner diameter of 1.91 m. It produces a magnetic field of 1.43 T. This is used in conjunction with the tracking detectors and is designed to measure the momenta of the charged particles passing through it. To compensate for the effect of this field on the electron beam, a high-field solenoid of 5 T is situated behind the RCAL.

Figure 4.05: Schematic outline of the ZEUS Calorimeter.
There is a second lower resolution calorimeter situated outside the UCAL called the Backing Calorimeter (BAC). The main purpose of this is to measure the energy leakage that has escaped through the UCAL, but it also serves as a yoke for the magnetic flux from the main solenoid. It consists of 7.3 cm thick iron plates.

There are also muon detectors; an inner set of chambers between the BAC and the UCAL, and an outer set, beyond the iron yoke. Each of these sets comprises three chambers, namely the Forward (FMUI/FMUO), Barrel (BMUI/BMUO) and Rear (RMUI/RMUO) Inner/Outer Muon chambers.

In the rear part of the detector, 7.5 m from the interaction point, stands a large iron scintillator which is used to reject events due to collisions involving atoms from the residual gas in the beam pipe. This is known as the *Vetowall*. Between 20 and 110 m from the interaction point in the forward direction can be found the Leading Proton Spectrometer (LPS) and the Forward Neutron Calorimeter (FNC) (at 105.9 m), which are designed to detect very low angle protons and neutrons, respectively, that have missed detection by the main ZEUS components by escaping down the beam-pipe. The Proton Remnant Tagger (PRT) is a lead scintillator, 5.1 m from the interaction point. There are two further lead scintillator calorimeters in the backward direction, at 34 m and 104 m from the interaction point, respectively. These are designed to measure outgoing electrons and photons at low angles in order to determine the luminosities of the beams, as well picking up photoproduction and radiative events.
4.2.3.1 Central Tracking Detector

The CTD is a cylindrical wire chamber located in the central core of the ZEUS detector, covering the polar angle range $15^\circ < \theta < 165^\circ$. It consists of nine superlayers, each consisting of eight sensor wires. The inner radius of the inner chamber is 16.2 m, the outer radius is 85 cm and the length is 2.41 m. Five superlayers have sensor wires parallel to the chamber axis (beam-pipe) and are known as axial superlayers. The other four superlayers have sensor wires at a small angle (~5°) to the main axis, which enables the $z$-coordinate of a particle track to be determined. These are called stereo superlayers. The chambers are filled with a gas mixture of argon (83%), carbon dioxide (5%) and ethane (12%) which has been bubbled through ethanol[33]. The advantage of this mixture is that it prolongs the life of the detector and adds to the safety, as opposed to a fifty-fifty mixture of argon and ethane which would give a higher resolution and less noise[34].

The CTD only detects charged particles passing through it. As the particle passes through the chambers, ion-electron pairs are created along its path. The electrons migrate towards the positive sensor wires at a speed of around 50 $\mu$m/ns and the positive ions migrate towards the negative sensor wires. After a signal amplification of around $10^4$, the signals are digitized using 8-bit ADCs. This process is known as a ‘hit’. For trigger purposes the inner three axial superlayers have a $z$-by-timing system installed which allows the $z$-coordinate of the hit to be determined by timing the arrival of the signal pulses.

The hit resolution in the $r - \phi$ plane is 200 $\mu$m (with a slight dependence on $\theta$). The $z$-resolution is about 2 mm using the axial/stereo layers and about 4 cm using the
z-by-timing mechanism. The relative resolution in transverse momentum for a particle passing all nine superlayers is given by:

$$\frac{\sigma(p_T)}{p_T} = 0.0058 p_T \oplus 0.0065 \oplus \frac{0.0014}{p_T}, \quad (4.04)$$

where $p_T$ is measured in GeV. The symbol, $\oplus$, indicates addition in quadrature. The first term on the right side of Equation 4.04 comes from the resolution in hit position, the second term comes from smearing from multi-scattering with the CTD itself and the third term comes from multi-scattering before the particle reaches the CTD.

### 4.2.3.2 Uranium Calorimeter

The Uranium Calorimeter (UCAL) surrounds the inner tracking system and solenoids. It is shown schematically in Figure 4.05 and the its angular coverage is given in Table 4.02. It is a *compensating* calorimeter which means that it responds equally to electrons (and photons) and hadrons (i.e., $e/h = 1 \pm 0.02$). Normally hadrons shower differently from electrons and photons as illustrated in Figure 4.06. This shows that an electron or photon, with the same energy as a hadron, produces more photon showering. Compensation for this difference is achieved by the use of *absorber* plates of depleted uranium and *activator* plates of plastic (polystyrene) scintillator, which are placed in alternating layers. The absorber plates are 3.3 mm thick, which corresponds to one radiation length$^{11}$, and are composed of 98.1% $^{238}$U, 1.7% Nb and $\sim$0.2% $^{235}$U. The plastic scintillator active plates are 2.6 mm thick. The calorimeter *cells* are made up of many alternating layers.

---

$^{11}$ One radiation length corresponds to the thickness of material which reduces the mean energy of an electron by a factor $e$ (the natural exponential constant).
consists of the EMC, which is about 25 interaction lengths electromagnetic showers. Each HAC is made up of one cell. The inner part of the towers RCAL and two 10 cm wide with a height varying between 2.2 and 4.6 m. An FCAL module is made up of towers and each tower is segmented into an electromagnetic calorimeter (EMC) and a hadronic calorimeter (HAC). Each EMC is made up of four 5 × 20 cm² cells in the FCAL and RCAL and two 10 × 20 cm² in the BCAL, which is sufficient to contain purely electromagnetic showers. Each HAC is made up of one cell. The inner part of the towers consists of the EMC, which is about 25 interaction lengths¹² deep. In the FCAL and BCAL, the outer part of the towers consists of two HAC sections (HAC1 and HAC2) whereas the RCAL only has one outer HAC section. Each HAC section is about two nuclear interaction lengths deep. This asymmetry reflects the different energy absorption requirements in the forward and backward directions due to the different proton/electron beam energies.

<table>
<thead>
<tr>
<th>Section of UCAL</th>
<th>Polar Angle Range</th>
<th>Pseudorapidity Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCAL</td>
<td>$2.2^\circ &lt; \theta &lt; 36.7^\circ$</td>
<td>$4.0 &gt; \eta &gt; 1.1$</td>
</tr>
<tr>
<td>BCAL</td>
<td>$36.7^\circ &lt; \theta &lt; 129.1^\circ$</td>
<td>$1.1 &gt; \eta &gt; -0.74$</td>
</tr>
<tr>
<td>RCAL</td>
<td>$129.1^\circ &lt; \theta &lt; 176.2^\circ$</td>
<td>$-0.74 &gt; \eta &gt; -3.4$</td>
</tr>
</tbody>
</table>

Table 4.02: Table showing angular coverage of the UCAL sections.

¹² One nuclear interaction length is the mean distance travelled before a nuclear interaction with the material.
Particles entering the calorimeter cells produced showers of particles that create UV light in the scintillator which is then read by 2 mm thick wavelength shifter bars attached to both sides of the module. Here photomultiplier tubes (PMTs) convert this light into an electrical signal. Each cell is read simultaneously by two PMTs, one on each side, which provides a more accurate positioning of the impact point within the cell plus some redundancy, as well as reducing the noise generated by the PMTs and its infrastructure.

The calorimeter is calibrated, one channel at a time, by using the natural radioactivity of the depleted uranium which gives a constant steady signal. This provides a calibration accuracy of around 1%. The PMTs are calibrated using light emission from the known light intensity of LEDs. The electronic readout chain is calibrated using test pulses.

The energy resolution given by the calorimeter for hadrons is given by:

\[
\frac{\sigma(E)}{E} = \frac{0.35}{\sqrt{E}} \oplus 0.02, \tag{4.05}
\]
and for electrons the resolution is given by: 
\[ \frac{\sigma(E)}{E} = \frac{0.18}{\sqrt{E}} \oplus 0.01 \]  

(4.06)

The angular resolution for particles detected is better than 0.1 mrad and the time resolution is better than 1 ns for energy deposits larger than 4.5 GeV. A more detailed description of the ZEUS calorimeter can be found in [35,36,37].

4.2.4 ZEUS Trigger System

Electron and proton bunches in the HERA ring cross in the ZEUS detector every 96 ns which corresponds to a frequency of around 10 MHz. The frequency that an interaction is recorded in the ZEUS detector is around 10–100 kHz and the vast majority of these interactions are background due to beam gas upstream of the interaction point as well as halo muon, cosmic rays and other background. The actual rate of useful ep collisions is around 3–5 Hz. In order to select out this useful data, ZEUS employs a sophisticated three level trigger system, logically called the First Level Trigger (FLT), Second Level Trigger (SLT) and Third Level Trigger (TLT). This is shown schematically in Figure 4.07[38].
4.2.4.1 First Level Trigger

The First Level Trigger is a purely hardware trigger system designed to bring the data rate down from 10 MHz to around 1kHz. Each component of the detector has its own FLT system which can make a decision on an event within 2 μs of it occurring. All the decision made up by these component FLTs go to the Global First Level Trigger (GFLT).
which then makes a decision on whether to accept or reject the event within about 4.4 \( \mu s \) of the event occurring. Events that are selected are passed on to the SLT.

### 4.2.4.2 Second Level Trigger

The Second Level Trigger is a software based trigger consisting of a network of transputers designed to lower the data rate down to around 100 Hz. Like the FLT, each ZEUS component has an individual SLT which feeds into the Global Second Level Trigger (GSLT)[39]. This then makes the decision on whether to accept or reject the event within about 3 ms of the original \( ep \) collision taking place. Events that are accepted by the GSLT are then passed to an Event Builder[40] which collects data from all the ZEUS components and compiles it into a single event record. This is then passed onto the Third Level Trigger.

### 4.2.4.3 Third Level Trigger

The Third Level Trigger is also a software based trigger that makes a decision on the event based on all the information about the event. It even includes some offline reconstruction code. It reduces the event rate down to a few Hertz. Events that are accepted by the TLT are written to tape via a fibre-link (FLINK) connection and made available for full offline reconstruction and data analysis[41]. Each event consists of about 100 kB of data.
5 Data Selection

Events used in the analyses of this thesis were selected from data collected by the ZEUS detector during various running periods (depending on the analysis) from 1996 – 2000. During the 1996 – 97 running period, HERA was colliding positrons ($E_e=27.5 \text{ GeV}$) with protons ($E_p=820 \text{ GeV}$) and about 38.6 pb$^{-1}$ was recorded. During the 1998 – 99 running period, the positron beam was replaced by an electron beam of the same energy and the proton beam energy was increased to 920 GeV. This data sample amounted to around 16.8 pb$^{-1}$. For the 1999 – 2000 running period, the electron beam was changed back to positrons and around 65.1 pb$^{-1}$ was recorded.

The data selected for these analyses all consisted of NC DIS events so the selection is similar for both of them. This selection is described in the following sections.

5.1 Identification of the Scattered Lepton

For NC DIS events to be selected, it is essential to identify the scattered electron$^{13}$ in the final state and that its position and momentum be measured accurately. The basis for identifying the scattered electron lies in the patterns of energy recorded in the UCAL. Electromagnetic showering differs in its topology to that of hadronic showering, particularly if the electron that leads to the electromagnetic shower is of sufficiently high energy and is well isolated from other energy deposits. Problems can occur in identifying the scattered electron under the following conditions:

$^{13}$ The term ‘electron’ is used to generically describe the scattered lepton, whether it be a positron or an electron.
(i) if the scattered electron has only a small amount of energy, the showering in the calorimeter can be similar to a hadronic shower of similar energy.

(ii) if the scattered electron ends up close to a hadronic shower in the calorimeter, it can be hard to determine which energy deposits belong to the hadrons and which belong to the electron itself, i.e., the electron shower can be partially obscured by the hadronic one.

(iii) some particles give false signatures, i.e., mimic the scattered electron. These include photons, $\pi^0 \rightarrow \gamma\gamma$ decays and $\eta$ particles whose energy deposits in the calorimeter are hard to distinguish from those of electrons. This problem can be avoided by using tracking detectors to make sure the particle is charged.

For the purposes of the current analyses, the electron finding is done using the neural network Sinistra[42]. This works by looking at islands of calorimeter cells, where an island is a group of cells merged together. By analyzing these islands for each event, the Electron Finder assigns each island a probability, $P$, of being the scattered electron according to certain criteria. Studies have shown that events having a probability of greater than 0.9 of containing an electron with energy greater than 10 GeV have higher efficiency and purity. Hence these criteria are used to select the required NC DIS events.
5.2 Reconstruction of Event Kinematic Variables

Two out of a number of event variables must be specified in order to completely describe the event kinematics. This arises from the fact that the event, in the QPM model, is essentially an electron-quark scatter (i.e., a two body process). As the ZEUS detector is almost completely hermetic, all the hadronic and scattered electron information is recorded and the event is over-constrained, so it is possible to determine the event variables in a number of ways, each of which is best suited to a specific kinematic region. The event variables are defined in Figure 5.01, which illustrates a QPM NC DIS scattering event. This diagram shows the four-momenta of the scattered particles before and after the event, as well as the angles through which they are scattered, all in the laboratory frame. The methods of determining the kinematic variables from the detector information are described in the following sections[43,44].

**Figure 5.01:** Kinematic variables of a DIS event.
5.2.1 Electron Method

The Electron Method only uses information from the scattered electron. This makes it particularly useful for fixed target experiments where this is the only information available. The measured quantities required are the initial and final electron energies, $E_e$ and $E_e'$, as well as the electron scattering angle, $\theta_e$, which illustrates the need for accurate and efficient electron finding and measurement. The Small angle Rear Tracking Detector (SRTD) was specifically installed for this purpose. The event variables for this method are then given by:

\[
x_e = E_e E_e' \left( \frac{1 + \cos \theta_e}{2E_p E_e - E_p E_e' (1 - \cos \theta_e)} \right),
\]

\[
y_e = 1 - E_e' \left( \frac{1 - \cos \theta_e}{2E_e} \right),
\]

\[
Q_e^2 = 2E_e E_e' (1 + \cos \theta_e).
\]

This method is most useful for low and medium $Q^2$ values (i.e. $Q^2 < 100 \text{ GeV}^2$).

5.2.2 Double Angle Method

The Double Angle Method[45] uses only angular variables, namely the electronic and hadronic angles, $\theta_e$ and $\gamma_h$. The usefulness of this method comes from the fact that ZEUS can measure the scattering angles of the particles to much higher precision than the energy because these quantities are independent of the energy scale of the calorimeter. The hadronic angle, $\gamma_h$, is given by:
The kinematic variables are given by:

\[ x_{DA} = \frac{E_e}{E_p} \cdot \frac{\sin \gamma_h + \sin \theta_e + \sin(\theta_e + \gamma_h)}{\sin \gamma_h + \sin \theta_e - \sin(\theta_e + \gamma_h)} \]  

\[ y_{DA} = \sin \theta_e \cdot \frac{1 - \cos \gamma_h}{\sin \gamma_h + \sin \theta_e - \sin(\theta_e + \gamma_h)} \]  

\[ Q_{DA}^2 = 4E_e^2 \cdot \frac{1 + \cos \theta_e}{\sin \gamma_h + \sin \theta_e - \sin(\theta_e + \gamma_h)} \]

In general, this method is the best for ZEUS analyses as it provides good reconstruction over a large part of the kinematic range accessible at HERA. However, in the medium \( Q^2 \) range (~10 – 100 GeV\(^2\)) and at large values of \( y \) (close to one), the electron method has been shown to be better.

### 5.3 Online Selection of NC DIS Events

Online selection of the NC DIS events refers to the application of the ZEUS trigger system discussed in Chapter 4. This has three levels (FLT, SLT and TLT). After the application of each trigger set, more information is known about the event and more time is available to make a decision as to whether to keep the event. Consequently, only loose requirements are placed on events by the FLT but quite strict criteria are applied at the TLT stage. Events that are kept are labelled with a tag called a ‘slot’, which indicates the type of event that it is or whether it has specific characteristics (e.g., the presence of a jet.
or two). Events selected for analysis generally are selected through a number of slots at each trigger level. They are usually selected through a logical OR method within each level and a logical AND between the levels.

5.3.1 First Level Trigger Selection

The Global First Level Trigger (GFLT) takes information principally from the Calorimeter First Level Trigger (CFLT) and tracking information from the CTD First Level Trigger. For the events relevant to this thesis, the FLT selects events which have properties common to those containing jets that are well separated from the proton remnant and have sufficiently high transverse energy. The following FLT slots were used:

- FLT slot 40 takes an event if there is more than 15 GeV of electromagnetic energy in the calorimeter,
- FLT slot 41 takes an event if there is more than 21 GeV of electromagnetic energy in the calorimeter,
- FLT slot 42 takes an event if there is a good track (i.e., one that is associated with the interaction point, is long enough for reconstruction in that it has at least five hits in each of the axial and stereo superlayer sections of the CTD) and one or more of the following is true:
  - the total energy in the calorimeter is greater than 15 GeV,
  - the total electromagnetic energy in the calorimeter is greater than 10 GeV,
  - the total electromagnetic energy in the BCAL is greater than 3.4 GeV,
the total electromagnetic energy in the RCAL is greater than 2.0 GeV,

• FLT slot 43 takes the event if there is a good track and the total transverse energy in the calorimeter is greater than 11.5 GeV.

The Vetowall, C5 and SRTD components of ZEUS are also used to reject beam-gas events and cosmic rays primarily based on timing information.

### 5.3.2 Second Level Trigger Selection

Events are selected by the SLT using information from all components. More time and information is available than at the FLT, allowing some reconstruction of the event to be done, for example calculation of the $E - p_z$ for the event. The proton beam travels in the $+z$ direction, and so $E - p_z$ is zero for the proton. The electron travels in the opposite direction, so $E - p_z$ for the electron is twice the beam energy (i.e., $2E_e = 55$ GeV). Hence the total initial event $E - p_z$ is 55 GeV and by conservation of energy and momentum, this must be the case after the interaction. Assuming that the ZEUS detector measures the complete event, then we can use this quantity to select NC DIS events. Another parameter that can be used by the SLT is the primary vertex position. The vertexing resolution can be poor\(^\text{14}\) but it is sufficient to remove beam-gas events and cosmic ray events. For the events selected in these analyses the following SLT slot was used:

• slot HPP01 takes the event if all of the following are true:
  
  o $E - p_z > 8$ GeV for the event or $\frac{p_z}{E} > 0.95$,

---

\(^{14}\)A ‘good’ vertex is defined as one that has at least two tracks associated with it and the $\chi^2$/n.d.f. for the fitting of the vertex is less than ten.
- there is at least one good track associated with the vertex.
- the vertex is reconstructed such that $|z_{\text{vertex}}| < 60$ cm or no vertex is reconstructed at all,
- the transverse energy is greater than 8 GeV.

### 5.3.3 Third Level Trigger Selection

For the TLT, the complete event information is available. All the CTD tracks are used to establish the location of the event vertex and a jet search is done using all the calorimeter information. For the events selected in these analyses the following TLT slots were used:

- **slot HPP01** is a high $E_T$ slot and takes the event if all of the following are true:
  - see HPP01 for SLT slot,
  - number of bad tracks is less than 6,
  - $8 < E - p_z < 75$ GeV.

- **slot HPP02** is an inclusive jet slot and takes the event if all of the following are true:
  - the vertex is reconstructed such that $|z_{\text{vertex}}| < 60$ cm or no vertex is reconstructed,
  - total $E - p_z < 75$ GeV.
  - number of bad tracks is less than 6,
  - at least one jet is reconstructed such that $E_{T,jet} > 10$ GeV and $\eta_{jet} < 2.5$.

- **slot HPP15** is a high $E_T$ dijet slot and takes the event if all of the following are true:
the vertex is reconstructed such that $|z_{\text{vertex}}| < 60$ cm or no vertex is reconstructed,

$E - p_T < 75$ GeV,

number of bad tracks is less than 6,

$\frac{p_z}{E} > 1.0$,

at least two jets are reconstructed such that $E_{T,jet} > 6$ GeV and $\eta_{jet} < 2.5$.

5.4 Offline Selection of NC DIS Events

5.4.1 Event Selection

Events surviving the trigger selection are written onto tape for analysis. The kinematic range chosen for the analyses is based on the following two cuts:

$$Q^2_{DA} > 125 \text{ GeV}^2 \quad \quad \quad \quad |\cos \gamma_h| < 0.65$$

The $\cos \gamma_h$ cut, was not used in selecting inclusive events for the second analysis, which uses the ratio of the cross section for high-$Q^2$ inclusive events and exclusive dijet events. This cut was used for selecting the dijet events and DIS events for the inclusive jets analysis. The purpose of this cut is to ensure good event reconstruction in the Breit frame [46,47]. Figure 5.02 shows a comparison of this variable calculated in the MC at both the hadron and detector levels. Figure 5.03 shows the ratio of this variable between the two levels. It can be seen that in the central region of this plot (that defines the cut) there is agreement to within 10% between the detector and hadron levels (as defined in Section
6.1), demonstrating that the detector has good acceptance in this region. These cuts which define the kinematic range, are also used to define the hadronic level.

For the purposes of the analyses in this thesis, further offline cuts were made to remove any potential background events, photoproduction events or events whose reconstruction is not reliable. These cuts are described below.

- Each event selected had to have at least one good scattered electron candidate with SINISTRA probability greater than 0.9 and energy greater than 10 GeV. This cut ensures that the electron finding efficiency is high and that photoproduction events are suppressed.
Isolation cuts are placed on the scattered electron. The total energy not associated with the electron candidate, within a cone of radius 0.8 units in the $\eta$-$\phi$ plane, has to be less than 10% of the total electron energy. This cut removes photoproduction events and also events in which part of a jet is misidentified for the scattered electron. Also, if the scattering angle for the electron, $\theta_e$ is within the range $30^\circ$ – $140^\circ$, then this cut of 10% of the total scattered electron energy was lowered to 2% - again, to remove events in which part of a jet is mistaken for the scattered electron.

**Figure 5.03:** Plot showing the ratio of $\cos \gamma_h$ evaluated at the hadron and detector levels in ARIADNE MC. The cuts and region of good reconstruction are shown.
• $40 < E - p_z < 60$ GeV. This cut removes background from non-$ep$ events (e.g., due to beam-gas collisions or cosmic rays) and photoproduction events. Also, it removes events that are not well constructed because of QED initial state radiation (ISR) that escapes down the beam pipe, i.e., radiation coming from the electron beam before the interaction takes place. These events have low overall $E - p_z$.

• $y_e < 0.95$. This cut removes events in which a photon in the FCAL is mistaken for a positron candidate.

• $\frac{P_T}{\sqrt{E_T}} < 3$ $GeV^{1/2}$, where $P_T$ is the total transverse momentum of all the final state particles and $E_T$ is their total transverse energy. These quantities are determined by summing over all the final state particles, labelled $i$:

$$P_T = \sqrt{\left( \sum E_i \sin \theta_i \cos \phi_i \right)^2 + \left( \sum E_i \sin \theta_i \sin \phi_i \right)^2}, \quad (5.08)$$

$$E_T = \sum E_i \sin \theta_i. \quad (5.09)$$

For a totally contained event, the total transverse momentum should be zero. However, due to the finite resolution of the calorimeter, the deviation of the transverse momentum scales approximately with transverse energy multiplied by the resolution. So by imposing this cut, events from cosmic rays and beam related background can be removed. Such events can leave a lot of energy on one side of the detector, relative to the beam pipe, leading to an overall imbalance in $P_T$ (i.e., deviation from zero).
• The location of the vertex for the event has to be consistent with \( ep \) collisions that are well contained within the calorimeter and CTD acceptances. This was achieved by imposing the cut \( |z_{\text{vertex}}| < 34 \text{ cm} \).

• The radius, \( R \), at which the scattered electron hits the RCAL has a cut imposed on it; \( R > 36 \text{ cm} \), where

\[
R = \sqrt{X^2 + Y^2}.
\] (5.10)

This avoids events where the RCAL acceptance region adjacent to the beam pipe is low.

5.4.2 Jet Selection

Jet finding was performed on groups of energy deposits within the calorimeter. The four-momenta of each of these deposits are established and boosted into the Breit frame before using the \( k_T \)-cluster algorithm to find the jets. Once found, the jet variables \( \{E_{T,\text{jet}}, \eta_{\text{jet}}, \phi_{\text{jet}}\} \) were established for each jet, in both the Breit and laboratory frames. The jet cuts were then applied, depending on the analysis in question.

5.4.2.1 Inclusive Jet Analysis Cuts

For the inclusive jets analysis the following cuts were applied with respect to jets:

• Before any corrections are applied, the pre-selection cut of \( E_{T,\text{jet}}^{L} > 3 \text{ GeV} \) was applied, where the superscript indicates the quantity in the laboratory frame.

• The event was removed from the data sample if either one of the following two conditions were satisfied:
- Any jet is found in the event with $E_{T,\text{jet}}^B > 5$ GeV and $\eta_{\text{jet}}^L < -2$. High energy jets in the backward direction are more likely to be photons from Initial State Radiation (ISR) (that have been detected by the BCAL) than jets and therefore fake.

- Any jet that is found in the event that is too close to the scattered positron, i.e., within one unit on the $\eta-\phi$ plane. In these cases the scattered electron can be too close to distinguish from the jet or the jet itself could be a radiated photon from the scattered electron, i.e., QED final state radiation (FSR). This can lead to uncertainties when boosting from the laboratory to the Breit frames.

The jets found in the data were then corrected for detector effects and also scaled by a factor determined from Monte Carlo studies, to account for uncertainties in the calorimeter energy scale (see Section 8.3.2 on jet energy corrections). Once corrected, the following cuts were applied to the jets:

$$E_{T,\text{jet}}^B > 8 \text{ GeV} \quad -2 < \eta_{\text{jet}}^B < 1.5$$

$$E_{T,\text{jet}}^L (\text{uncorr}) > 2.5 \text{ GeV} .$$

The cut in the laboratory frame on the uncorrected jet energy is to ensure that jets are sufficiently collimated and have sufficient energy to be accurately reconstructed in the calorimeter. The equivalent energy cut in the Breit frame is chosen so as to minimize the theoretical uncertainties from the NLO calculations. The cut on $\eta_{\text{jet}}$ in the Breit frame is designed to only use parts of the calorimeter where jets can be well reconstructed and
avoid regions where fake jets are common. This cut is also done in the Breit frame to reduce the theoretical uncertainties. The upper limit is equivalent to a cut of $\eta < 2.5$ in the laboratory frame as can be seen from the plot in Figure 5.04.

For the 1998 – 2000 ZEUS data sample, once all these cuts had been applied at the event and jet levels, a total of 26650 jets were found from a total of 20018 events.

**5.4.2.2 Dijet Analysis Cuts**

This analysis requires that each event have two and only two jets, such that $E_{T,jet}^B > 5$ GeV. The selection of events and reconstruction of the jets follows exactly the same method as in the inclusive jet analysis. Once the events and jets have been selected,
the events are only kept as dijet events if the highest $E_T^{\text{jet}}$ is greater than 8 GeV. The reason for this asymmetric jet cut lies in the theoretical calculations of exclusive dijet events. It turns out that if there is a symmetric cut (i.e., both jets are subject to the same minimum $E_T$ cut), then the calculated cross sections are infra-red sensitive in the region where both jets have an $E_T$ close to the minimum. This infra-red sensitivity comes from the fact that there is not enough three-body phase space available in the vicinity of the $E_{T,\text{jet}1} = E_{T,\text{jet}2}$ region for the compensation between the real and virtual contributions to take place. This results in an unphysical small peak in the calculated differential cross section with respect to $E_{T,\text{jet}}$ close to the minimum $E_T$[48].
6 Monte Carlo Event Simulation

6.1 Purpose of Monte Carlo Generation

Monte Carlo has the following uses for the analyses in this thesis:

(i) It is used to simulate the effects of the detector. This is necessary before comparisons between experimental results and theoretical predictions can be made. The latter predictions are usually based on results calculated at the parton or hadron level. The detector attempts to measure this but the result is dependent on its response to the particles resulting from the event. This is known as the detector acceptance. For this reason, a concrete understanding of the detector is required in order for these responses to be modeled and information about the input particles established. This is done by using the MC to simulate events at the hadron level which are then fed into a detector simulation software package\(^{15}\) to produce simulated data. By comparing the hadron level MC to the detector level, correction factors can be established that allow measurements made in the real data to be corrected back to the hadron level. These results at the hadron level can then be compared to theoretical predictions.

(ii) NLO predictions do not generally account for QED radiation. This is radiated photons that are emitted by the incoming electron (ISR) and the outgoing scattered electron (FSR), as well as internal virtual loops (see

\(^{15}\) ZEUS simulation software is based on the GEANT 3.13 and is collectively known as AMADEUS.
Figure 6.01). However, this radiation needs to be taken into account as it has a direct effect on the measured cross sections in the data. Correction factors to account for this discrepancy are obtained by comparing the MC generated predictions at the hadron level which have been obtained with this QED radiation switched both on and off.

Figure 6.01: Diagram of QED radiative effects (ISR, FSR and virtual loops).

(iii) Monte Carlo events are generally calculated using pQCD diagrams but only up to leading order (LO). More accurate theoretical predictions can be established using programs that calculate diagrams up to next-to-leading order (NLO). Predictions from such programs are generally made at the parton level, whereas the data are usually corrected to the hadron level (see (ii)). For this reason, the MC events are used to obtain correction factors (hadron-parton corrections) that can be used to correct NLO predictions to the hadron level. Results are normally presented at the hadron level. This is because correction factors applied to the data are not
expected to improve significantly. It is possible to apply hadron-parton corrections to the hadron level measurements and present these for comparison directly with the NLO predictions at the parton level. However, should the NLO improve, it is easier to use them to correct calculated predictions than apply them to published data, which may or may not already contains such corrections.

(iv) Monte Carlo is used to compare experimental results with theoretical predictions. This allows a judgment to be made on the validity of those predictions and to tune any floating parameters used in making them, so that the prediction fits the data.

Three MC generator programs are used in the analyses relevant to this thesis. They are summarized in Table 6.01. RAPGAP[49] is shown as well as ARIADNE[50] and LEPTO[51], both of which are interfaced to HERACLES[52] via DJANGO[53]. HERACLES is used to generate either NC or CC $ep$ interactions, using PDF parameterizations with radiative corrections optionally applied. These can be single photon emissions from the electrons or self energy corrections including the complete set of one-loop weak corrections. ARIADNE is used to simulate QCD cascades using the Colour Dipole Model (CDM)[54] and LEPTO uses LO electroweak cross-sections, first order QCD matrix elements, parton showers[55] and the Lund String hadronization model[56].
<table>
<thead>
<tr>
<th>Program Name</th>
<th>Description</th>
<th>Showering Method</th>
<th>Hadronization Method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HERACLES</strong></td>
<td>Generates NC and CC $ep$ interactions, optionally with ISR/FSR radiative corrections using a single photon emission, self energy corrections and the complete set of one loop weak corrections.</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>ARIADNE</strong></td>
<td>Simulates QCD cascades implementing the color dipole model (CDM).</td>
<td>CDM</td>
<td>Lund String Model</td>
</tr>
<tr>
<td><strong>LEPTO</strong></td>
<td>Simulates DIS $ep$ scattering based on LO electroweak cross sections implementing matrix elements plus parton showering (MEPS).</td>
<td>MEPS</td>
<td>Lund String Model</td>
</tr>
<tr>
<td><strong>DIANGO</strong></td>
<td>Interface between HERACLES and LEPTO/ARIADNE to give complete $ep$ events.</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>RAPGAP</strong></td>
<td>Generates DIS $ep$ events, including diffractive DIS and LO direct and resolved processes.</td>
<td>MEPS</td>
<td>-</td>
</tr>
<tr>
<td><strong>JETSET</strong></td>
<td>Implements the Lund String Model for ARIADNE and LEPTO generators.</td>
<td>-</td>
<td>Lund String Model</td>
</tr>
</tbody>
</table>

Table 6.01: Table showing summary of Monte Carlo programs.

### 6.2 Implementation of Monte Carlo

Monte Carlo is generated in stages which correspond to how the event process is modeled. This is described in Section 1.2 and in more detail here, in relation to the MC event simulation. Figure 6.02 shows the details of a typical event process.
6.2.1 Hard Scattering Sub-process

The hard scattering can be factorized into a convolution between the partonic interaction (i.e., the electron-quark scatter) and the non-perturbative parton distribution function. The former can be calculated using pQCD, while the latter cannot. The hard scatter is generated in the MC using LO matrix elements and the PDF is treated as an external input. At the end of this stage of the event generation, there is a scattered electron, a proton remnant and a number of partons resulting from the hard scatter.

6.2.2 Parton Showering

In DIS, the quarks involved in the hard scatter can emit partons both before and after the interaction, giving rise to a shower of initial and final state partons. This is illustrated in Figure 6.03. This serves to evolve the hard scattering result into the final state hadrons.
and as a model for incorporating higher order QCD effects. These higher order effects can be hard and time consuming to calculate from Feynman diagrams. There are a number of phenomenological models used to simulate the parton showering; LEPTO uses the Matrix Elements plus Parton Showering model (MEPS)[55] and ARIADNE uses the Colour Dipole Model (CDM)[54].

![Diagram of parton showering.](image)

**Figure 6.03**: Diagram of parton showering.

### 6.2.2.1 Matrix Element Plus Parton Showering Model

This involves the splitting of the gluon into $q\bar{q}$ or gluon pairs, or quarks radiating gluons as shown in Figure 6.03. This proceeds by the use of the DGLAP description of the branching process as described in Section 1.6.2. Showering is controlled by the virtuality of the partons involved in the splitting. These parton virtualities come in three main categories; ‘on-shell’ ($m^2 = 0$), ‘off-shell, space-like’ ($m^2 < 0$) and ‘off-shell, time-like’ ($m^2 > 0$). Two different types of showering are allowed to occur. Before the interaction,
space-like showers occur where a parton that is close to being on-shell splits off time-like
partons (which in turn can initiate time-like showers) and becomes increasingly space-
like until it participates in the hard interaction. After the interaction, the result is a parton
that is close to being on-shell or time-like in virtuality. In this case, this parton splits off
time-like partons until it is fully on-shell. The outcome of all this initial and final state
showering (calculated using the DGLAP equations) is a set of partons that are all on-
shell. The initial and final state QCD radiation is combined with the Matrix Element
calculation. Interference terms between the initial and final state radiation are excluded
and QCD emission by the proton remnant is not considered either.

6.2.2.2 Colour Dipole Model for Showering

The Colour Dipole Model is based on the theory of classical dipole radiation. The struck
quark involved in the hard scatter carries a colour charge and the proton remnant has the
corresponding anti-colour so together they form a colour dipole. All the radiation then
results from this dipole, whether seen as initial or final state radiation. This is illustrated
for the $e^+e^-$ scattering in Figure 6.04. The $ep$ scattering case is just a modification of this principle. The main difference is that the proton remnant cannot be considered a point-like particle whereas the quark can. This is taken into account by allowing only a fraction of the proton’s remnant momentum to take part in the gluon emission, which reduces the available phase space.

The parton showering proceeds via a series of gluon emissions, which create further dipoles, which in turn led to further, softer gluon emission. Boson-Gluon-Fusion (BGF) (see Section 1.2.1) cannot be taken into account by this method alone. To overcome this problem, a matching procedure is used for the first emission. The initial dipole between the struck quark and the proton remnant either emits a gluon according to the LO matrix element (the case described above) or emits the antiquark partner to the struck quark according to the BGF matrix element. This leaves a quark and an antiquark outside the proton, each of which can give rise to a dipole with the proton remnant, which can radiate gluons as described above.

The CDM takes into account the QCD coherence effects that are included in the Parton Showering model by ordering the final state emissions according to angle. It should be noted that the non-point-like nature of the proton remnant is taken into account in this model, unlike the Parton Showering model.

### 6.2.3 Hadronization

After the parton showering stage, a large number of quarks and gluons result, all of which are coloured and so cannot be detected as free particles. These partons must undergo a
process of combining into colourless particles (i.e., hadrons) through a process of hadronization. This cannot be described using pQCD methods and so phenomenological models are required to describe the process. The two most common hadronization models used are the Lund String Model, as used in ARIADNE and LEPTO and implemented by the software package JETSET\cite{57}, and the Cluster Model\cite{58}, which is used by HERWIG\cite{59}. These two models are described below. The output from these is considered to be the hadron level MC that is used to obtain QED and hadron-parton corrections for the data.

\textbf{6.2.3.1 The Lund String Model}

This model for hadronization uses the formation of colour strings between $qq$ pairs. As any given $qq$ pair separates, the colour force string between them stretches out and the force between them increases linearly. As the force increases the potential energy stored in the string also increases until the point at which it is energetically favourable to ‘break’ the string and create an extra quark-antiquark pair, $q\bar{q}$ at the ends of its fragmented parts (a process known as \textit{fragmentation}). This is illustrated in Figure 6.05. This gives two quark-antiquark pairs $q\bar{q}$ and $\bar{q}q$, each of which has a shortened colour string between them. These two pairs can now evolve independently in the same way until the all the pieces of string are short enough for the particles to be considered mesons and the invariant masses of the $qq$ pairs are below some threshold. Baryons are created in the same process through the formation of a diquark-quark group instead of a quark-antiquark pair. As gluons and sea quarks are introduced, the string structure becomes
even more complicated[60]. However, the final state is a collection of colourless hadrons which represents the hadronic final state. This model is infra-red safe with respect to soft or collinear gluon emission.

**Figure 6.05:** Diagram of Lund String Model for hadronization.

### 6.2.3.2 The Cluster Model

The Cluster Model[58] for hadronization is based on gluons splitting non-perturbatively into $qq$ pairs and $qqqq$ groups[61]. This is illustrated in Figure 6.06. Quarks and antiquarks that are close to each other combine into colour-singlet groups. These have a universal mass spectrum which is steeply falling at high mass. These groups then break up into hadrons. If a group is too light to split into two or more hadrons, it becomes a single final state hadron of some flavour. Those groups that are big enough, then decay isotropically into two or more hadrons, whose flavour depends on the density of available
states for each quark flavour, with appropriate quantum numbers. Masses can be shifted by exchanging momenta with nearby neighbour clusters. This method has the inherent advantage of being a way to generate transverse momenta for the final state particles. It also has fewer parameters that need tuning.

![Diagram of Cluster Model for hadronization.](image)

**Figure 6.06**: Diagram of Cluster Model for hadronization.

### 6.2.3.3 The Hadronic Final State

Once the hadronization process has been modeled, it is then necessary to define the hadronic final state. This necessity arises from the fact that the hadrons coming out of the hadronization process are not stable in that they do not have infinite lifetimes. Ideally the hadronic final state is defined as consisting of those hadrons which are stable enough to reach the detector and be measured. Some of those hadrons will have emerged directly from the fragmentation process at the interaction point. However, some will be the
products of hadrons which have decayed en route from the interaction point. To
distinguish which particles will reach the detector, a critical lifetime, $\tau_c$ is introduced,
which is based on the time taken for a particle to reach the detector. Particles with a
lifetime longer than this critical lifetime are considered stable and part of the hadronic
final state. Particles with lifetimes less than this value, have their decay products
considered as part of the hadronic final state. The value of the critical lifetime chosen for
these analyses was $\tau_c = 10^{-11}$ s [46,62]. Using this definition, it should be noted that not
all the particles in the hadronic final state are hadrons. Leptons and photons are also the
decay products of short-lived hadrons and make up part of the final collection of particles
arriving in the detector.

6.2.4 Detector Simulation

For the detector level MC, a simulation of the ZEUS detector is used, which takes the
hadron level MC, obtained after the hadronization process and produces the detector level
MC. This detector level MC is simulated data, which are equivalent to real data that
would be seen in ZEUS if those final state hadrons were detected. The detector
simulation is based on GEANT 3.13[63] and includes the response of all the detector
components, the effect of inactive material within the detector and a simulation of the
ZEUS trigger. These simulated data are processed and reconstructed using the same
offline procedures as real data and stored on tape in the same manner.
7 Next-to-Leading Order Calculations

NLO calculations are needed for two reasons with respect to the analyses in this thesis. Firstly, they can be used to make predictions that can be compared to data measurements in order to evaluate the validity of the QCD theory behind those calculations. Secondly, through this comparison, the value of the free parameter, $\alpha_s$ (as well as information about the PDF) can be established, due to the sensitivity of the result to the value of this parameter. These calculations are available in the Breit frame for inclusive and multijet events as well as the mean subjet multiplicity for jets in the laboratory frame. These predictions are only available at the parton level and need to be corrected to the hadron level before comparison with data measurements can be made.

NLO calculations are done in a similar way to the LO MC described in Chapter 6. Only pQCD diagrams are calculated (omitting the phenomenological models that follow) to give the parton level cross sections needed. The ‘events’ calculated refer to final states given by the different contributions that are being studied. Calculations require the use of MC methods. Hard scattering processes based on the relevant Feynman diagrams are calculated by NLO QCD diagrams, which contain the matrix elements required. This is done using the program DISENT[64], which is described later in this chapter.

7.1 A Comparison of NLO with Leading Order Monte Carlo

NLO calculations and LO MC are both used in the analyses described in this thesis but for different purposes. These differences are summarized below.
• Both methods use matrix elements in their calculations but whereas LO MC uses only the matrix elements from LO diagrams, the NLO calculations include NLO diagrams, which considerably increases their precision. This means that at the parton level, NLO is better suited to providing predictions with which to compare data measurements. However, phenomenological methods such as parton showering and hadronization can be used in LO MC to simulate the higher order effects as well as soft partonic radiation. This means that LO MC provides predictions at the hadron level as well as the parton level. The hadron level is also well suited for input into detector simulations to provide detector level predictions. This makes the LO MC well suited to providing corrections for hadronization as well as for detector effects. This is described in more detail in Chapter 6.

• As LO MC employs phenomenological methods for parton showering and hadronization, there are a large number of interdependent parameters that need to be tuned through comparison with data measurements. NLO predictions are only available at the parton level and depend purely on pQCD calculations. As a result, the only tunable parameters are those quantities that cannot be calculated (i.e., $\alpha_s$ and the PDF). NLO predictions also have free parameters that are introduced in order to make the calculations possible, namely the factorization and renormalization constants, $\mu_F$ and $\mu_R$ (see Chapter 1). The small number of such free parameters in NLO calculations makes it easier to estimate the uncertainties in their choice and is an advantage in the extraction of $\alpha_s$ and the PDF. This is not
possible in LO MC due the fact that this is only a model and has a large number of parameters.

- For NLO, the calculations start at any required order of $\alpha_s$, so that orders that give zero cross section can be ignored. This is not the case for LO MC, which always starts at order zero in $\alpha_s$, and uses parton showering to estimate higher order contributions. This makes NLO calculations much better suited to multijet cross section calculations.

- Leading order MC programs are generally used as event generators, which can randomly discard events so as to get an equal weight for all events. This is not possible for NLO, as there is no upper limit for the weight of the events. Also, as NLO uses cancellation of divergences through positive and negative weight diagrams, it is numerically less stable than LO MC. This is described in the next section.

### 7.2 DISENT

This program is used for providing NLO pQCD predictions. It has been widely used to provide such predictions for analyses done at HERA[46,47,62] and has proved to give reliable results for the quantities studied there. It works through generating configurations based on each term in the perturbative expansion, for partons in the initial and final states. Each configuration has an associated weight and is convoluted with the chosen PDF from which a cross section is calculated. A jet search is performed using the $k_T$-cluster algorithm and the same cuts in phase space are applied as for the hadron level
(see Chapter 5). If accepted, the weight of the configuration is then added to the relevant cross section bin, which leads to a final cross section after all the events have been processed. Matrix elements are calculated within the program and IR divergences are removed using the Dipole Formalism[65] and the Subtraction Method[66]. Details of the calculation are given below.

### 7.2.1 Parton Level Cross Section

The NLO cross section is given by the perturbative series that has been truncated after the second term:

$$\sigma^{NLO} = \sigma^{(0)} + \sigma^{(1)},$$  \hspace{1cm} (7.01)

where the numbered superscripts on the right hand side indicate the LO term and the NLO correction term respectively. The LO term, \( \sigma^{(0)} \) is obtained by integrating the fully exclusive Born cross section, \( d\sigma^B \), over the total phase space given by the physical observable. If the final state involves \( n \) partons, with four momenta, \( p_k \), such that \( (k = 1,\ldots,n) \), then at LO:

$$\sigma^{(0)} = \int_n d\sigma^B,$$  \hspace{1cm} (7.02)

where the Born cross section is given by:

$$d\sigma^B = d\Phi^{(n)}\left(\{p_k\}\right) |M_n(\{p_k\})|^2 F_j^{(n)}(\{p_k\}),$$  \hspace{1cm} (7.03)

where \( d\Phi^{(n)} \) represents the full phase space and \( M_n \) represents the tree level QCD matrix which gives \( n \) final state partons. The function \( F_j^{(n)} \) represents the physical quantity
under study (i.e., the jet cross sections for the jets that result from the final state partons). $\sigma^{(0)}$ is calculated within the DISENT program, using MC and numerical techniques.

For the NLO correction, $\sigma^{(1)}$, the following equation is used:

$$\sigma^{(1)} = \int_n^{n+1} d\sigma^R + \int_n^{n} d\sigma^V + \int_n^{n} d\sigma^C ,$$  

(7.04)

where $d\sigma^R$ represents the real corrections, with $n+1$ partons in the final state, $d\sigma^V$ represents the virtual one-loop corrections, with $n$ partons in the final state and $d\sigma^C$ is the collinear subtraction term that results from having to redefine bare parton densities into factorization scale dependent PDFs. These are given by:

$$d\sigma^R = d\Phi^{(n+1)} |M_{n+1}(\{p_k\})|^2 F_{J}^{(n+1)}(\{p_k\}) ,$$  

(7.05)

$$d\sigma^V = d\Phi^{(n)} |M_{n}(\{p_k\})|^2_{\text{loop}} F_{J}^{n}(\{p_k\}) ,$$  

(7.06)

where $|M_{n}(\{p_k\})|^2_{\text{loop}}$ represents the renormalized QCD amplitude evaluated in the one-loop approximation giving a final state with $n$ partons:

$$|M_{n}|^2_{\text{loop}} = M_{n} \cdot (M_{n}^{\text{loop}})^* + M_{n}^{\text{loop}} \cdot (M_{n})^* ,$$  

(7.07)

where $d\Phi^{(n+1)}$ represents the full phase space and $M_{n+1}$ represents the tree level QCD matrix, which gives $n+1$ final state partons.

The problem with the calculation of these real and virtual cross sections ($d\sigma^R$ and $d\sigma^V$) is that soft and collinear singularities arise, caused by partons becoming collinear with another initial or final state parton. The solution to this lies in the regularization of the phase-space integrals on the right hand side of equation 7.04 in a number of space-time dimensions, $d = 4 - 2\varepsilon$, which allows one to express these divergences simply in terms of
single and double poles, i.e., $\frac{1}{\epsilon}$ and $\frac{1}{\epsilon^2}$. To get a finite NLO correction, $\sigma^{(1)}$, these poles are required to cancel between the virtual and real contributions and the collinear term, $d\sigma^C$. This cancellation is only guaranteed for the calculation of observables that are so called infra-red safe and allow initial state collinear singularities to be factorized.

### 7.2.2 Subtraction Method

The subtraction method depends on the equation:

$$\sigma^{(1)} = \int_{n+1} [d\sigma^R - d\sigma^A] + \int_n d\sigma^V + \int_{n+1} d\sigma^A + \int_n d\sigma^C, \quad (7.08)$$

where the term $d\sigma^A$ represents a ‘fake’ cross section that has been subtracted from the first term in equation 7.04 and added back in at the end. This term is required to satisfy the following conditions:

- any given observable has to be obtained in a way that is independent of the jet observable under consideration. The Dipole Formalism described below provides a recipe for constructing $d\sigma^A$ that is completely process independent.

- it has to be a close approximation of $d\sigma^R$ in that it has to have the same pointwise singular behaviour as $d\sigma^R$ in $d$ dimensions. Due to this, $d\sigma^A$ is a local counter-term to $d\sigma^R$ and it is possible to follow the limit $\epsilon \rightarrow 0$ in equation 7.08.

- it must be analytically integrable in $d$ dimensions over the one-parton subspace leading to the soft and collinear divergences. This means that the last three terms on the right hand side of equation 7.08 can be written
\[ \sigma^{(1)[n]} = \int_n^\infty \left[ d\sigma^V + \int_1^\infty d\sigma^A + d\sigma^C \right]_{\varepsilon=0}. \quad (7.09) \]

The middle term on the right hand of equation 7.09 results in a \( \varepsilon \)-pole contribution that combines with the other two terms and cancels all the divergences. This leaves a finite part in the limit \( \varepsilon \to 0 \), which is the integrand that represents the \( n \)-parton kinematics part of the cross section. The resultant NLO parton level cross section is then given by:

\[ \sigma^{NLO} = \sigma^{(0)[n]} + \sigma^{(1)[n+1]} + \sigma^{(1)[n]} . \quad (7.10) \]

This can be implemented using MC techniques, which generates ‘events’ weighted appropriately, with \( n \) and \( n+1 \) partons.

The important aspect of this method is the choice of the term \( d\sigma^A \). DISENT relies on the dipole formalism as described below.

### 7.2.3 Dipole Formalism

This is used to construct the fake cross section, \( d\sigma^A \) in a fully process independent manner by using the factorizing properties of gauge theories. Factorization theorems are used to write cross sections as a contraction of the Born cross section with universal soft and collinear factors. However, care is required, as these theorems are only valid in exact singular limits, so extrapolating away from these limits can be problematic (e.g., in the treatment of momentum conservation). The double counting of soft and collinear divergences in the overlapping regions must also be avoided, e.g., where a gluon is both soft and collinear to another parton. The dipole formalism provides a recipe that
overcomes these difficulties in a simple manner. It is used to relate the tree-level matrix elements with $n+1$ partons, $M_{n+1}$ to $M_n$. It can be written in the following way:

$$|M_{n+1}(p_1, \ldots, p_{n+1})|^2 = |M_n(\vec{p}_1, \ldots, \vec{p}_n)|^2 \otimes V_{ij} + \ldots,$$

(7.11)

where $V_{ij}$ represents the dipole splitting functions. These are universal, process-independent singular factors that depend on the momenta and quantum numbers of the $n$ partons in the tree-level matrix element $|M_n|^2$. The dots on the right represent contributions that are not singular when $p_i \cdot p_j \to 0$ and the symbol $\otimes$ denotes the colour and helicity correlations. The $\vec{p}_1, \ldots, \vec{p}_n$ represent modified four-momenta, which are defined starting from the original $n+1$ parton momenta in such a way that the $n$ partons in $|M_n|^2$ are physical (i.e., they are on-shell and energy-momentum conservation is obeyed exactly).

Equation 7.11 gives a formula for the real matrix element, $|M_{n+1}|^2$, in a single formula, that is valid for arbitrary processes and in all of its singular limits. These limits are approached smoothly and double counting of overlapping soft and collinear singularities is avoided. Through the precise definition of the modified four-momenta, exact factorization of the $n+1$ parton phase space can be obtained, which allows the universal splitting function to be integrated. By itself, this factorization is not enough to give the fake cross section term, $d\sigma^A$, because the phase space depends on the jet observable under consideration. However, as the modified four-momenta are physical, this can be overcome. The fake cross sections are constructed by adding dipole contributions to the right side of equation 7.11. For each contribution, the jet observable is calculated from
the corresponding $n$ modified parton four-momenta, not from the original $n+1$ parton four-momenta. This can be done without anything being known about the jet observable in question, as these four-momenta are fixed during the analytical integration.

### 7.2.4 Summary of the DISENT Method

The first step is to generate an $n$-parton event in the phase space region at leading order, $d\Phi^{(n)}$, and give it a weight $|M_n|^2$. This ‘weighted event’ is then analyzed in an appropriate user routine, which applies cuts to it and should it survive, it is placed in an appropriate cross section histogram bin.

The NLO correction is made up of the two contributions shown in equation 7.10, which correspond to $n$ and $n+1$ parton kinematics. These terms are derived through the use (i.e., addition and subtraction) of the fake cross section term, $d\sigma^A$, which is found using the Dipole Formalism. Unlike the original real and virtual contributions, these terms are separately finite and integrable in four space-time dimensions. The last of these terms, $d\sigma^{(1)}$ has a structure identical to the LO partonic cross section but with the Born term replaced by the finite sum of the virtual matrix elements in $d\sigma^V$ plus the collinear counter term $d\sigma^C$ plus the analytical integral of the dipole contributions in $d\sigma^A$. The other correction term (second term of 7.10), that involves $n+1$ parton kinematics and is obtained by subtracting the fake cross section, $d\sigma^A$, from the real NLO correction (see equation 7.08), has the following form:
\[ d\sigma^{(1)(n+1)} = \int_{n+1} d\Phi^{(n+1)} \cdot \left\{ |M_{n+1}([p_k])|^2 F_{J}^{(n+1)}([p_k]) - \sum_{ij} \{|M_{n}([p_k])|^2 \otimes V_{ij} F_{J}^{(n)}([p_k])\} \right\} \]

(7.12)

The term in the curly bracket represents an effective matrix element that is finite and integrable in four space-time dimensions.

The algorithm does not require knowledge of the parton densities during the MC integration. The partonic cross section can be expressed in terms of the partonic momentum fraction, which can then be convoluted with the PDF after the MC integration.
8 Corrections to the Measured Cross Sections

8.1 Cross Section Definitions

The results of the analyses described in this thesis are presented in the form of differential cross section plots, which show the comparison between the calculated theoretical cross sections and those measured in the data. Based on this comparison, the value of the strong coupling constant, $\alpha_s$, can be determined. The term cross section indicates the probability of such an event happening. For example, the differential cross section for a dijet event in a particular $Q^2$ bin indicates the probability of a dijet event occurring with a value of $Q^2$ in the range of that particular bin. The experimental definition of cross section is given by

$$\sigma = \frac{N}{L}, \quad (8.01)$$

where $N$ is the number of inclusive or dijet events (in the case of the ratio analysis) or jets (in the case of the inclusive jets analysis) and $L$ is the luminosity of the colliding beams. However, a cross section measured directly by counting events or jets in the detector requires the application of correction factors to the raw result. These correction factors are necessary for a number of reasons and are obtained from Monte Carlo simulations. The details of the reasons for these correction factors are given in Section 6.1.

The final hadron level measured cross section is then given by the following equation.

$$\sigma_{had}^{meas} = C_{QED} \cdot C_{Z^0} \cdot C_{acc} \cdot \sigma_{data}^{data}, \quad (8.02)$$
where \( C_{\text{QED}} = \frac{\sigma_{\text{MC had, noQED}}^{\text{MC}}}{\sigma_{\text{had, QED}}^{\text{MC}}} \), is the correction applied to account for the effects of initial and final state QED radiation. This is described in Section 6.1 and illustrated in Figure 6.02.

\( C_{Z^0} = \frac{\sigma_{\text{MC noZ}}^{\text{MC}}}{\sigma_{Z^0}^{\text{MC}}} \) is the correction applied to account for weak boson exchange instead of a photon. The \( C_{Z^0} \) correction is significant for high \( Q^2 \) events where a \( Z^0 \) boson can be exchanged instead of a virtual photon.

\( C_{\text{acc}} = \frac{\sigma_{\text{had}}^{\text{MC}}}{\sigma_{\text{det}}^{\text{MC}}} \) is the correction that accounts for inefficiencies due to the detector. For example, these maybe caused by inactive material used to construct the detector components. The numerator in this expression represents the cross section calculated at the hadron level in the MC and the denominator represents the cross section calculated at the detector level in the MC after the hadron level has been put through a detector simulation. The hadron and detector levels in the MC are illustrated in Figure 6.02.

For a cross section that is differential in a variable, \( X \) (where \( X \) could be \( Q^2 \), jet transverse energy, etc.), the formula used in (8.01) is modified:

\[
\frac{d\sigma_{\text{meas}}^{\text{had}}}{dX} = C_{\text{acc}} \cdot C_{\text{QED}} \cdot C_{Z^0} \cdot \frac{N}{L \cdot \Delta X},
\] (8.03)

where \( N \) is the number of events in a bin of width \( \Delta X \) as illustrated in Figure 8.01.
8.2 Control Plots

The Monte Carlo used to make these corrections relies on leading order QCD calculations to simulate the events. This MC must describe the data to within the statistical uncertainties in order to account for inefficiencies, migrations and purities in the data. In order to assess the validity of the MC description of the data and to help determine the value of required cleaning cuts, control plots are used. Figure 8.02 shows the control plots for event variables important in selecting the DIS events used for the inclusive jets analysis. In these plots, the black points represent ZEUS data and the histograms represent the corresponding simulated data obtained using the Ariadne MC and the ZEUS detector simulation program. Each plot is normalized to cover an area of one unit so that it is the shape that is compared. It can be seen that there is very good agreement between the detector level MC and the data distributions. The one exception is the distribution in $E - p_z$, which arises from an incomplete understanding of the calorimeter energy scale with respect to the scattered electron. This quantity is not important in the selection of events and so this discrepancy is not regarded as critical.
Figure 8.03 shows the control plots for the jet variables in the Breit frame and Figure 8.04 shows the control plots for the same jet variables in the laboratory frame. Again, each of these plots shows good agreement between data and the MC simulation. In conclusion, this MC is suitable for acquiring the necessary corrections in order to obtain accurate inclusive jet cross section measurements.

8.2.1 Control Plots for Inclusive Jets Analysis

Figure 8.02: Event control plots for inclusive jets sample (in all plots, the area normalized to one):
(a) $Q_{D\Lambda}^2$
(b) $y_{D\Lambda}$
(c) Corrected energy of the scattered electron
(d) $E - p_z$ for scattered electron
(e) Angle of scattered electron, $\theta_e$
(f) Cosine of hadronic scattering angle, $\cos\gamma_h$.  

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Figure 8.03: Jet control plots for variables in the Breit frame for the inclusive jet analysis.

Figure 8.04: Jet control plots for variables in the laboratory frame for the inclusive jet analysis.
8.2.2 Control Plots for Ratio Analysis

The term ‘ratio analysis’ refers to the analysis that uses the ratio between inclusive DIS events and dijet event cross sections. Figure 8.05 shows control plots for those event variables important to selecting the inclusive DIS events required for this analysis. As for the inclusive jets, these plots show how the data (black points) and the MC (histograms) compare directly. Each plot is normalized to cover an area of one unit, so that it is the shape that is compared. Figure 8.06 shows control plots for those event variables important to selecting the dijet DIS events used. Figure 8.07 shows control plots for those variables important in selecting the jets. Once again, each of these plots shows good agreement between data and MC simulation to within the statistical uncertainties, meaning that the ARIADNE MC sample is suitable for this analysis. As before, the one exception is the distribution in \( E - p_z \), which for the same reasons as the inclusive jet distributions, shows some discrepancy. Again, this is not considered critical to the analysis.
Figure 8.05: Event control plots for inclusive event sample:
(a) $Q_{DA}^2$  (b) $y_{DA}$  (c) Corrected energy of the scattered electron
(d) $E - p_z$ for scattered electron.
Figure 8.06: Event control plots for dijet event sample:
(a) $Q^2_{DA}$ (b) $y_{DA}$ (c) Corrected energy of the scattered electron
(d) $E - p_z$ for scattered electron.
Figure 8.07: Jet control plots for dijet sample in Breit frame.
8.3 Jet Energy Corrections

8.3.1 Main Corrections

For much the same reason acceptance corrections are required to correct for the inefficiencies of the detector, jet energy corrections are also needed, in particular to correct for the imperfections in the calorimeter and for loss of energy by the constituent particles of the jet as they pass through inactive material (e.g., the beam pipe) before reaching the calorimeter. In principle, this could be covered by the acceptance corrections themselves, which compare the number of jets at detector level with the number of jets at hadron level and produce a correction factor accordingly. However, it is important to know the precise energy of these jets in order to prevent migrations of the jets into the wrong energy bin for those differential cross section measurements. It should be noted that the scattered electron also suffers from energy losses due to inactive material but as the Double Angle method is used to reconstruct its energy, the uncertainties are not significant and energy corrections are not necessary. Again, a sample of ARIADNE MC is used and the procedure is performed in the laboratory frame. This is because the correction factors are dependent on the part of the detector being used, in particular on the pseudorapidity in the laboratory frame and not on the kinematics of the event.

The method for establishing the correction factors is described below.

- A high statistics sample of MC is used to provide a sample of jets at the hadron and detector levels.
• For the hadron level the same cuts as in the main inclusive jets analysis are used to select both jets and events. At the detector level, the same cuts are also used to select events but the jet cut is lowered to 2.5 GeV and only jets in the pseudorapidity range [-2.5, 2.5] in the laboratory frame and the pseudorapidity range [-2.0, 2.0] in the Breit Frame are used. As in the main analysis, events with jets close to the scattered positron or with possible fake jets in the backward region are completely removed from the sample.

• Jets are then matched at the hadron and detector levels. Each hadron jet is matched to the closest detector level jet in \( \eta - \phi \) space and kept in the final sample if the separation, \( \Delta R \), in \( \eta - \phi \) space is less then one, where
\[
\Delta R = \sqrt{(\eta_{\text{had}} - \eta_{\text{det}})^2 + (\phi_{\text{had}} - \phi_{\text{det}})^2}.
\]

• For each hadron-detector jet pair, the energy of the hadron jet, \( E^{L}_{T, \text{jet}}(\text{true}) \), is taken as the true energy, and the energy of the detector jet, \( E^{L}_{T, \text{jet}}(\text{det}) \), is taken as the reconstructed energy. This sample of jet pairs is then divided into fourteen sets depending on which pseudorapidity region they lie in. These regions are given in Table 8.01.
For each region, a correlation plot is generated between the transverse jet energies at the hadron and detector levels. An example of one of these plots is given in Figure 8.08. Each plot shows the mean value of the transverse energy of the detector level jets in that detector region, $\overline{E_{T,\text{jet}}^{L,\text{det}}}$, as a function of the transverse energy of the jets at the hadron level, $E_{T,\text{jet}}^{L,\text{true}}$. 

Table 8.01: Pseudorapidity regions for determining jet energy corrections.
Linear fits of the form, \( E_{T,jet}^{L} (det) = c + m \cdot E_{T,jet}^{L} (true) \) were then performed on each of the 14 plots. The cut on energy at 2.5 GeV biases the plot at the lower end and so the first few points are excluded from the fit. In most cases, the points do not lie on a single straight line but two or more disjointed straight lines (i.e., the fit parameters vary slightly with jet energy). Hence, each plot is divided into sections and the overall list of fit parameters depend not only \( \eta_{jet}^{L} \), but also on \( E_{T,jet}^{L} (det) \).

**Figure 8.08:** Example plot used to find fit parameters for jet energy corrections.
Once the fit parameters have been established the corrected jet energy is given by the formula:

\[ E_{T,jet}^L (corr) = \frac{E_{T,jet}^L (det) - c}{m} \]  

(8.04)

A correction factor, \( r_{corr} \) is then applied to the transverse energies in the Breit frame where:

\[ r_{corr} = \frac{E_{T,jet}^L (corr)}{E_{T,jet}^L (det)} \quad \text{and} \quad E_{T,jet}^B (corr) = r_{corr} \cdot E_{T,jet}^B (uncorr) \]  

(8.05)

In all regions the linear fit is extremely good with a \( \chi^2 / \text{n.d.f.} \) much less than one. Once the jet energy correction factors have been applied, the jets are then subjected to the cuts described in Section 5.4.2 and the surviving ones are those that make up the final sample. Table 8.02 shows the extracted fit parameters that were used in the analyses.
<table>
<thead>
<tr>
<th>Detector Region (η_{jet} range)</th>
<th>E_{T,jet}^{L} (det) /GeV</th>
<th>Fit Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-2.0, -1.0]</td>
<td>&lt; 9.854</td>
<td>c = -0.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.79</td>
</tr>
<tr>
<td></td>
<td>&gt; 9.854</td>
<td>c = -0.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.84</td>
</tr>
<tr>
<td>[-1.0, -0.75]</td>
<td>&lt; 12.58</td>
<td>c = 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.69</td>
</tr>
<tr>
<td></td>
<td>&gt; 12.58</td>
<td>c = 0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.71</td>
</tr>
<tr>
<td>[-0.75, -0.5]</td>
<td>&lt; 13.52</td>
<td>c = 0.336</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.69</td>
</tr>
<tr>
<td></td>
<td>&gt; 13.52</td>
<td>c = -0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.81</td>
</tr>
<tr>
<td>[-0.5, -0.25]</td>
<td>&lt; 21.16</td>
<td>c = -0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.88</td>
</tr>
<tr>
<td></td>
<td>&gt; 21.16</td>
<td>c = -0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.85</td>
</tr>
<tr>
<td>[-0.25, 0.0]</td>
<td>&lt; 17.15</td>
<td>c = -0.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.82</td>
</tr>
<tr>
<td></td>
<td>&gt; 17.15</td>
<td>c = -0.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.87</td>
</tr>
<tr>
<td>[0.0, 0.25]</td>
<td>&lt; 19.26</td>
<td>c = -0.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.86</td>
</tr>
<tr>
<td></td>
<td>&gt; 19.26</td>
<td>c = -0.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.87</td>
</tr>
<tr>
<td>[0.25, 0.5]</td>
<td>&lt; 17.35</td>
<td>c = -0.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.84</td>
</tr>
<tr>
<td></td>
<td>&gt; 17.35</td>
<td>c = -1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.89</td>
</tr>
<tr>
<td>[0.5, 0.75]</td>
<td>&lt; 18.94</td>
<td>c = -0.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.84</td>
</tr>
<tr>
<td></td>
<td>&gt; 18.94</td>
<td>c = -1.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m = 0.90</td>
</tr>
</tbody>
</table>
8.3.2 Jet Energy Scale Corrections

In addition to the main jet energy corrections, an additional correction is needed to account for the difference in the absolute energy scale of the jets in the data and MC, which further reduces the uncertainty in the jet energies to around 1% or better. The

Table 8.02 (and previous page): Fit parameters used for jet energy corrections.

| [0.75, 1.0]       | < 12.49 | c = 0.053 | m = 0.73 |
|                  | 12.49 – 21.96 | c = -0.72 | m = 0.84 |
|                  | 21.96 – 29.45 | c = -1.33 | m = 0.87 |
|                  | > 29.45   | c = -0.86 | m = 0.87 |
| [1.0, 1.25]      | < 17.59  | c = -0.37 | m = 0.78 |
|                  | > 17.59  | c = -1.38 | m = 0.84 |
| [1.25, 1.5]      | < 17.02  | c = -0.65 | m = 0.84 |
|                  | 17.02 – 26.54 | c = -1.04 | m = 0.86 |
|                  | > 26.54  | c = -1.62 | m = 0.89 |
| [1.5, 1.75]      | < 24.89  | c = -0.59 | m = 0.87 |
|                  | > 24.89  | c = 0.95  | m = 0.90 |
| [1.75, 2.0]      | < 26.60  | c = -0.41 | m = 0.90 |
|                  | > 26.60  | c = -0.67 | m = 0.92 |
| [2.0, 3.0]       | < 27.52  | c = -0.94 | m = 0.98 |
|                  | > 27.52  | c = -2.00 | m = 1.01 |
method for acquiring this correction, involves studying an observable that is sensitive to that energy scale in both the data and MC. The observable chosen, denoted $r$, is the ratio of the jet transverse energy to the event transverse momentum calculated using the Double Angle method:

$$r = \frac{E_{T,\text{jet}}}{p_{T,DA}}$$

where

$$p_{T,DA} = \frac{Q_{DA}^2}{y_{DA}} (1 - y_{DA})$$

Since the denominator in Equation 8.06 is calculated using the Double Angle method, it is independent of the energy scale of the calorimeter (to first order) and so the difference in this variable between the data and MC is due to the jet energy in the numerator. Hence the factor $r_{\text{data}}/r_{\text{MC}}$ can be used to correct the jet energies in data. It should be noted that the main jet energy corrections discussed in the previous section are to correct the jet for losses due to inactive material between the hadrons coming out of the event and the detector and is done by directly comparing the jets at hadron and detector level in the MC only. These jet energy corrections are applied in both the data and MC when obtaining acceptance corrections.

The jet samples used to get these corrections are obtained from the same data set and MC as the main analysis. However, to maximize the jet sample and therefore the precision in obtaining these corrections, the event selection is modified.

- Since the corrections depend on the region of the detector, jet finding is done in the laboratory frame. Cuts of $E_{T,\text{jet}} > 10$ GeV and $\eta_{\text{jet}} < 2.5$ are used.

- The event cut on $\cos\gamma_h$ is removed. The Breit frame is not used and there is no need to restrict the sample of jets.
• The jet energies are not corrected. The data and MC are treated equally.

• The cleaning cuts are not used (i.e., where events are removed due to the possible presence of fake jets, and/or a jet is too close to the scattered positron).

• Events are kept if they contain just one jet over 10 GeV and there is no second jet above 5 GeV. This is because we need to compare $p_{T,DA}$ with the jet energy. As $p_{T,DA}$ is calculated from event variables, it is necessary to have only one jet.

Once the jet sample was obtained, the mean value of the observable, $r$ is plotted as a function of $E_{T,jet}$ for both the data and MC in five different regions of the detector determined by the jet pseudorapidity (similar to the main jet energy corrections discussed in the previous section). This plot is shown in Figures 8.09.

The jet energy scale correction is then obtained for each pseudorapidity range using a linear (flat) fit through the central four bins where the statistics are highest. These corrections are shown in Table 8.03. They are inverted and used as a correction factor for each individual jet in the data only.
Table 8.03: Jet energy scale corrections obtained from Figure 8.08.

<table>
<thead>
<tr>
<th>Pseudorapidity Range</th>
<th>Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-2.0, 0.0]</td>
<td>0.989</td>
</tr>
<tr>
<td>[0.0, 1.0]</td>
<td>0.995</td>
</tr>
<tr>
<td>[1.0, 1.5]</td>
<td>0.989</td>
</tr>
<tr>
<td>[1.5, 2.0]</td>
<td>1.01</td>
</tr>
<tr>
<td>&gt; 2.0</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Figure 8.09: Ratio of $r_{\text{data}}/r_{\text{MC}}$ for each region of the detector.
8.4 Other Corrections

8.4.1 Acceptance Corrections, Jet Purity and Efficiency

Acceptance correction factors are required to compensate for the inefficiency of the detector (e.g. due to inactive material used in the construction of the detector). These acceptance factors are found using the MC and applied on a bin-by-bin basis to the cross section plots, as described in Section 8.1. Underlying these correction factors are the efficiency and purity of the jet finding. The purity, $P$, of the jet sample in a particular bin is given by the following equation:

$$ P = \frac{\text{no. of matched jets at detector and hadron level}}{\text{total number of detector level jets}}. \quad (8.07) $$

The efficiency, $E$, of the jet finding is given by the following equation:

$$ E = \frac{\text{no. of matched jets at detector and hadron level}}{\text{total number of hadron level jets}}. \quad (8.08) $$

The ratio $P/E$ is equivalent to the ratio of the number of hadron level jets to the number of detector level jets which (given the luminosity is the same for both) is equivalent to the acceptance correction given in equation 8.02. The purity and efficiencies for the inclusive jets analysis cross section bins are shown in Figure 8.10. Ideally, the purity and efficiencies would be close to 100%. In practice, they are less but it is important that they are reasonable in order to keep the systematic uncertainties under control. Both the purities and efficiencies range between 60 and 80% for each distribution which is sufficient for the analyses to be performed.
This section shows plots illustrating the size of the correction factors used, obtained from the MC simulation. The correction factors for the inclusive jets analysis are shown in Figures 8.11 (single differential cross sections) and 8.12 (double differential cross sections).

**8.4.2 Corrections Plots**

This section shows plots illustrating the size of the correction factors used, obtained from the MC simulation. The correction factors for the inclusive jets analysis are shown in Figures 8.11 (single differential cross sections) and 8.12 (double differential cross sections).
Figure 8.11: Correction factors used in the inclusive jets analysis:
(a) $Q^2$  (b) $E_{T,jet}^a$  (c) $\eta_{jet}^a$

Top: acceptance correction for detector inefficiency.
Upper middle: correction for QED effects.
Lower middle: correction factor between hadron jets and parton jets in NLO.
Bottom: correction for electroweak effects (i.e., $Z^0$ exchange boson).
Figure 8.12: Correction factors used in the inclusive jets analysis (double differential cross sections in bins of $Q^2$). Details as in Figure 8.11.
The acceptance correction factors for these distributions are all less than 20%, which is good enough for the analyses to be performed. The correction factors required for QED radiation are less than 5% and the correction factors for weak boson exchange are less than 1%.

Figure 8.13 shows the correction factors obtained for the inclusive event sample used for the ratio analysis. Figure 8.14 shows the correction factors obtained for the dijet sample used for the ratio analysis. For the inclusive event sample the correction factors are similar to the inclusive jets sample. However, for the dijet sample, the acceptance correction factors are higher at around two. The other correction factors are similar to the inclusive jets sample.

![Correction Factors for Inclusive Q^2 Bins](image)

**Figure 8.13:** Correction factors used for the inclusive events sample for the ratio analysis. Details as in Figure 8.10.
In conclusion, all the correction factors for each sample are reasonable and considered acceptable for the relevant analysis.
9 Cross Section Results

In this chapter, the final results are presented in the form of NC DIS differential cross section plots. For the inclusive jets analysis, plots are presented for differential cross sections with respect to $Q^2$, $E_{T,jet}^B$ and $\eta_{jet}^B$. For the ratio analysis, plots are presented for differential cross sections with respect to $Q^2$ (both inclusive and dijet events) and $E_{T,jet}^B$ (for dijet events). A plot showing the ratio of dijet to inclusive events, with respect to $Q^2$ is also presented. The measurements are compared to NLO theoretical calculations based on pQCD. First, an explanation of the systematic uncertainties is given, followed by a discussion of the NLO calculations with associated theoretical uncertainties.

9.1 Systematic Uncertainties

Precision measurements of cross sections require a detailed understanding of the systematic uncertainties. This refers to a set of uncertainties that arise from assumptions that are not completely valid, generally due to incomplete knowledge of the detector or some imperfections in the analysis method. Many such uncertainties arise from the cuts used to select events and jets. The Monte Carlo used to provide corrections has to model these cuts exactly. If, at the detector level, some events are lost or gained due to migrations at the cut boundary, which does not model the data exactly, then a systematic uncertainty due to that cut will occur. That is why it is important that control plots show reasonable agreement between the data and detector level MC. Also, if the MC does not model the jet finding efficiency or purity well, larger systematic uncertainties can arise.
To check the size of these uncertainties, the source must be identified and then the analysis repeated with a slight modification corresponding to a variation in the assumption that was made. A summary of the sources of systematic uncertainty considered, and the size of the uncertainty that they lead to, is given below.

9.1.1 Common Systematic Uncertainties

The two analyses featured in this thesis are both based on jet finding algorithms in the Breit frame in NC DIS events. As a result, many of the sources of systematic uncertainty are common to both. These are described below.

- Jet energy scale uncertainty: section 8.3.2 describes how the uncertainty in the jet energy is reduced to around 1% for jets with transverse energy of 10 GeV or greater. However, even a 1% uncertainty in the energy of the jets due to the calorimeter, translates to an uncertainty of around 5% for the jet cross sections, both in the inclusive and dijet samples. This can be verified by scaling the energies of the jets in the MC samples by 1% for jets of $E_{T,\text{jet}} > 10$ GeV and 3% for jets of $E_{T,\text{jet}} < 10$ GeV (where the resolution of the calorimeter is poorer), and repeating the analysis. This represents the largest source of uncertainty and is strongly correlated between the histogram bins in which the measurement was made. For this reason it is shown as a separate (green) shaded band on the cross section plots presented later in the chapter.
• Calorimeter energy scale: this affects the reconstructed value of $E - p_z$, which is used to select DIS events. To assess this uncertainty, the cuts on $E - p_z$ were varied from [40, 60] to [35, 65] GeV. The effect of moving the upper cut value made no significant difference to the cross sections, but the variation in the lower cut led to a small uncertainty in the cross sections of 2–3%. The electron energy, as measured in the calorimeter, was also varied by 1% in the MC to reflect the uncertainty in this measurement. This scaling was done after the electron candidate had been selected by SINISTRA. This led to a very small uncertainty in the cross sections, less than 0.5%.

• Electron finder: to estimate the uncertainties introduced by the SINISTRA electron finder, an alternative was used, known as the EM finder. This led to an uncertainty in the cross sections of less than 2%.

• MC generator: to estimate the leading order model dependent uncertainties introduced through using the ARIADNE MC, which depends on the Colour Dipole Model for parton showering, a different MC model was used to obtain the acceptance correction factors, namely LEPTO-MEPS, which depends on the Matrix Elements and Parton Showering model. Using this MC model reduces the cross sections by up to 2%.

• Cut on $y_c$: this cut is used to select DIS events by removing events that may contain a fake positron candidate in the FCAL. To estimate the uncertainty associated with this cut, it was lowered from 0.95 to 0.9. Shifting the placement of this cut reduces the cross sections by less than 0.1%. 

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- Vertex cuts: the cut on the primary vertex used to distinguish between DIS events and beam-gas or cosmic ray events was changed from $|z_{\text{vertex}}| < 34$ cm to 50 cm. The resulting uncertainty was less than 0.1%.
- Luminosity: the uncertainty in the beam luminosity was estimated to be 1.6%.
- Trigger chain: the TLT configuration was changed from that described in Section 4.2.4.3, where events are selected on criteria related to jets, to a selection based on high-$Q^2$ events only. The uncertainty introduced by this change was less than 0.1%.

A summary of these systematic uncertainties is shown in Table 9.01 below.

<table>
<thead>
<tr>
<th>Systematic Uncertainty</th>
<th>Systematic Shift</th>
<th>Typical Size of Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet energy scale</td>
<td>±1% for $E_{T,\text{jet}}^B &gt;10$ GeV</td>
<td>±5% for $E_{T,\text{jet}}^B &gt;10$ GeV</td>
</tr>
<tr>
<td></td>
<td>±3% for $E_{T,\text{jet}}^B &lt;10$ GeV</td>
<td>±10% for $E_{T,\text{jet}}^B &lt;10$ GeV</td>
</tr>
<tr>
<td>Electron energy scale</td>
<td>±1%</td>
<td>±0.5%</td>
</tr>
<tr>
<td>$E - p_z$</td>
<td>[40,60] $\rightarrow$ [35, 65] GeV</td>
<td>$^{+2%}_{-3%}$</td>
</tr>
<tr>
<td>Inelasticity, $y_e$</td>
<td>0.95 $\rightarrow$ 0.9</td>
<td>less than $-0.1%$</td>
</tr>
<tr>
<td>$z_{\text{vertex}}$</td>
<td>[-34,34] $\rightarrow$ [-50,50] cm</td>
<td>less than ±0.1%</td>
</tr>
<tr>
<td>MC Generator</td>
<td>ARIADNE $\rightarrow$ LEPTO</td>
<td>$-2%$</td>
</tr>
<tr>
<td>Luminosity</td>
<td>–</td>
<td>$\sim 1.6%$</td>
</tr>
<tr>
<td>Electron Finder</td>
<td>Sinistra $\rightarrow$ EM Finder</td>
<td>±2%</td>
</tr>
<tr>
<td>Trigger Chain</td>
<td>Jets $\rightarrow$ High $Q^2$</td>
<td>less than ±0.1%</td>
</tr>
</tbody>
</table>

Table 9.01: Summary of systematic uncertainties.
9.1.2 Systematic Uncertainties for Inclusive Jets Analysis

A summary of the systematic uncertainties specific to the inclusive jets analysis is given below.

- Uncorrected jet energy cut: this cut in the laboratory frame was set at 2.5 GeV for the central measurement. To estimate the systematic uncertainty related to this cut, it was first removed entirely and then set at 4.0 GeV. The size of this uncertainty was typically very small at less than 1%.

- Cut on jet pseudorapidity: this cut was set at 2.0 for the central measurement and was intended to remove events with fake jets due to radiated photons. To estimate the systematic certainty, this cut was set at 1.5 and 3.0. The size of this uncertainty was typically 1%.

9.1.3 Systematic Uncertainties for Ratio Analysis

The purpose behind taking the ratio of the inclusive event cross section to that of the dijet cross section (as a function of $Q^2$) was to offset some of the systematic and theoretical uncertainties common to both. As a result, the systematic uncertainties related to event selection variables are expected to be smaller.

A summary of the systematic uncertainties specific to the ratio analysis is given below.

- Jet energy cuts: the differential cut on jet energy in the Breit frame was designed to avoid some difficulties in the NLO calculations related to the phase space close
to the cut boundary. The second highest energy jet cut was set at 5 GeV and the highest energy jet cut was set at 8 GeV for the central measurement. To estimate the uncertainty related to these cuts, it was placed in turn at 6.5(4.0) and 9.5(6.5) GeV for the (second) highest jet. The size of this uncertainty was typically 3(4)%, for the upper(lower) cut.

9.2 Theoretical Uncertainties

The NLO predictions needed in order to compare with the data measurements presented in this chapter were done using the program DISENT, which is discussed in detail in Chapter 7. As well as calculating the required differential cross sections for comparison with the data and extraction of $\alpha_s$, it is also necessary to assess the uncertainties associated with those calculations. These uncertainties come from the following sources.

(i) Uncertainty due to the PDF: the PDF is extracted using real data measurements as described in Section 2.3.3. The uncertainties in the PDF arise directly from the experimental data used in the PDF fit. These can be assessed naïvely by repeating the NLO calculation using an alternative PDF set or by using all the PDF uncertainty sets available for the PDF parameterization being used.

(ii) Renormalization scale uncertainty ($\mu_R$): this parameter was introduced due to the cut off, at next-to-leading order in the perturbation series used in the calculation. If the theoretical calculation could be carried out to all possible
orders in $\alpha_s$, then there would be no need to introduce this arbitrary parameter. However, since the calculation is only performed to next-to-leading order, this parameter is introduced to compensate for the missing orders. The value of this parameter is typically chosen to be a suitable hard scale related to the analysis in question\(^\text{16}\). For the inclusive jet analysis, the renormalization scale was chosen to be the transverse energy of the most energetic jet, $\mu_R = E^B_{T, j1}$.

For the ratio analysis, this is not appropriate as there are no jets in the inclusive event cross section calculation. Instead, the virtuality of the exchange boson is used, $\mu_R = Q$. It is necessary to assess the uncertainty associated with the choice of this parameter. By convention, this is done by repeating the NLO calculations, with the value of this parameter scaled by a factor of two each way. This uncertainty is the most significant, typically around 5%.

(iii) Factorization scale uncertainty ($\mu_F$): this parameter is used to separate processes that happen inside the proton from those outside. For both analyses, $\mu_R$ was set to the value of the virtuality of the exchange boson, $Q$. To estimate the uncertainty in this choice, the NLO calculation is repeated with the value of this parameter scaled by a factor of two each way. This uncertainty proves not to be significant, typically less than 1%.

\(^\text{16}\) There are a number of methods of choosing a value for the renormalization constant, $\mu_R$. The one chosen here is based on a physical scale relevant to the processes involved, the so-called Physical Scale Argument. Other methods commonly used are (i) The Principle of Minimum Sensitivity\([67,69]\), (ii) Method of Effective Charges\([68]\) and (iii) The Brodsky-Lepage-Mackenzie Method\([69]\).
Hadron-parton corrections: these are computed using the leading order MC models. Uncertainties in these corrections arise from differences in the MC models used to obtain them. They are also obtained from a finite number of events. As this is a statistical process, there will be a statistical uncertainty associated with the corrections obtained (but this is small as the statistics of the generated MC sample is very high, close to 15 million). Overall this uncertainty is small (less than 0.5%).

9.3 Cross Sections

The cross section results presented in this section are differential cross sections, normalized to bin width and plotted at the centre of gravity of the bin (i.e., the weighted mean of the entries within that bin). The cross sections are presented at the hadron level (i.e., the data points are corrected for detector effects, QED radiative effects and $Z^0$ exchange). The parton level NLO calculations are corrected to the hadron level and compared directly to the data measurements.

9.3.1 Inclusive Jets Analysis Results

The differential jet cross sections as a function of $Q^2$, $E^{B}_{T,jet}$ and $\eta^{B}_{jet}$ are shown in Figure 9.01. The description of the plots can be found in the captions. The upper portion of each plot shows the direct comparison between the measured cross section (data points) and the theoretical prediction (solid line). The lower portion shows the ratio of data to theory
and includes a theoretical error in the form of a shaded band. The horizontal central axis represents perfect agreement, i.e. data measurement and theory are equal. For agreement between the data and the theory, the error bars on the data points must overlap the theoretical uncertainty band. It can be seen from Figure 9.01 that both the differential cross sections with respect to $Q^2$ and $E_{T,\text{jet}}^B$ fall steeply with increasing $Q^2$ or $E_{T,\text{jet}}^B$. It can also be seen that the measured cross sections and the theoretical cross sections are consistent within the uncertainties. In the case of the differential cross sections with respect to $Q^2$ and $E_{T,\text{jet}}^B$ this consistency is shown over four orders of magnitude.

Figure 9.02(a) shows differential cross sections as a function of $E_{T,\text{jet}}^B$ in bins of $Q^2$. These are not quite double differentials as they are not normalized to the width of the $Q^2$ bins. Figure 9.02(b) shows the ratio between the data and the NLO prediction for these cross sections. The bin boundaries for these cross sections, as well as the measured cross sections, are given in Table 9.02. It can be seen from Figure 9.02 that the same decline in cross section is seen with respect to $E_{T,\text{jet}}^B$ for each region of $Q^2$. It can be noted that the data measurements are higher than the theory for the lowest two $Q^2$ plots and that the jet energy uncertainty band is broadest here. However, consistency is seen between the measured and theoretical cross sections with the exception of the highest $E_{T,\text{jet}}^B$ point in the highest $Q^2$ region. The statistics in this region are extremely low which can lead to large fluctuations and consequently the uncertainties here may be underestimated. However, consistency is still present at the two sigma level.
Figure 9.01:
The inclusive jet differential cross section as a function of (a) $Q^2$ (b) $E_{T,jet}^\mu$ (c) $\eta_{jet}^\mu$.
The data points are shown as solid dots and are presented at the hadron level. The solid line shows the NLO theoretical prediction corrected to the hadron level. The lower plot shows the ratio of data to NLO prediction. The inner error bars represent the statistical uncertainty and the outer bars the total experimental uncertainty (statistical and systematic uncertainties added in quadrature). The green (solid) shaded band represents the systematic uncertainty in the jet energy scale.
Figure 9.02: (a) The inclusive jet differential cross section as a function of $E_{T, jet}^B$ in bins of $Q^2$.
(b) Ratio of data measurement to NLO prediction.
Details as in Figure 9.01.
Table 9.02: Binning for inclusive jet analysis cross sections and cross section results.

<table>
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<th>Variable</th>
<th>Bin Number</th>
<th>Bin Boundaries</th>
<th>Bin Width</th>
<th>Cross Section</th>
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</thead>
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<td>1.12 pb/GeV$^2$</td>
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<td>250</td>
<td>0.370</td>
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<tr>
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<td>3</td>
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<td>125.1</td>
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</table>

9.3.2 Ratio Analysis Results

The differential cross section for inclusive events as a function of $Q^2$ is shown in Figure 9.03. The differential cross sections for dijet events as a function of $Q^2$, $E_{T,\text{jet}}^B$, and $E_{T,\text{jet}2}^B$ are shown in Figure 9.04. The descriptions can be found in the captions. Figure 9.05 shows the ratio between the inclusive event cross section and the dijet event cross section.
section, both as a function of $Q^2$. The bin boundaries for these cross sections, as well as the measured cross sections, are given in Table 9.03.

<table>
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<tr>
<th>Variable</th>
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<th>Bin Width</th>
<th>Dijet Cross Section</th>
<th>Inclusive Event Cross Section</th>
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<td>1.117 pb/GeV$^2$</td>
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Table 9.03: Binning for ratio analysis cross sections and cross section results.

The plots in Figures 9.03 and 9.04 indicate the same decreasing behaviour in differential cross section with respect to $Q^2$ and $E_{T,jet}^B$ as seen with the inclusive jet cross sections. This is the case for both the dijet and inclusive events. For the dijet cross sections with respect to $E_{T,jet}^B$, there is a maximum at low $E_{T,jet}^B$ which is caused by the lack of phase space close to the cut boundaries, as described in Section 5.4.2.2. It can be also noticed that in this region the theoretical uncertainty, indicated by the shaded band on the lower part of the plots, is much higher and rises sharply with decreasing $E_{T,jet}^B$. Again,
consistency between the measured and calculated cross sections is evident to within the uncertainties over four orders of magnitude.

Figure 9.05 shows the ratio of differential cross sections with respect to $Q^2$ between dijet and inclusive events. This ratio is used in the extraction of $\alpha_s$ because a number of the systematic and theoretical uncertainties common to both parts of the ratio cancel out, potentially reducing the overall uncertainties. The equation for this ratio is shown in Figures 9.03 to 9.05, with the part corresponding to the plot highlighted. It can be seen in Figure 9.05 that this ratio increases with increasing $Q^2$, indicating a greater fraction of events with increasing $Q^2$ that contain exactly two jets.

**Figure 9.03:** The inclusive event differential cross section ($\sigma_{\text{tot}}$) as a function of $Q^2$. Details as in Figure 9.01.
Figure 9.05 also shows that the jet energy scale uncertainty associated with this ratio decreases with increasing $Q^2$. There is a minimum at the central point where the uncertainty nearly vanishes. This central point could be considered to be an anomaly but it should be noted that this uncertainty is itself subject to statistical uncertainties which
are increased due to the use of this ratio. The statistical uncertainties on the jet energy scale uncertainty are greater than its variation within this $Q^2$ region.

In summary, the consistency between the cross section results measured in the data and those calculated using pQCD theory is sufficient to confirm the validity of the pQCD theory. No significant discrepancies are observed. This consistency allows the use of these results to extract values of the QCD parameter, $\alpha_s(M_Z)$, using the comparison between the data and theoretical measurements. The results of this extraction are described in the next chapter.

Figure 9.05: Ratio ($R_{2+1}$) of inclusive event cross section to dijet event cross section as a function of $Q^2$. Details as in Figure 9.01.
10 Extraction of $\alpha_s(M_Z)$

10.1 Central Values of $\alpha_s(M_Z)$ and Related Uncertainties

The method for extracting a value of the QCD parameter, $\alpha_s(M_Z)$, from the experimental data and theoretical predictions is detailed below.

(i) Multiple sets of theoretical calculations were done, one for each of a range of fixed values of $\alpha_s(M_Z)$. For the MRST99 PDF set, three values of $\alpha_s(M_Z)$ are available; $\alpha_s(M_Z) = 0.1125, 0.1175$ and $0.1225$. This was the main set used in these analyses as it allowed direct comparisons to be made with previous analyses that used this PDF set[62]. The CTEQ4 PDF set was also used as a cross check and this has five values of $\alpha_s(M_Z)$ available; $\alpha_s(M_Z) = 0.110, 0.113, 0.116, 0.119$ and $0.122$.

(ii) The result of these calculations was that for each differential cross section experimental data point, taken as a function of the variables being used (i.e., $Q^2$ and $E_{T,\text{jet}}^B$), there are three (using MRST99) theoretical points (all corrected to the hadron level). The three theoretical points are used to parameterize the dependence of the differential cross section, $\frac{d\sigma}{dX}$ (where $X = Q^2$ or $E_{T,\text{jet}}^B$) on $\alpha_s(M_Z)$, according to the functional form:

$$\frac{d\sigma}{dX}[\alpha_s(M_Z)] = C_{1,i} \cdot \alpha_s(M_Z) + C_{2,i} \cdot \alpha_s(M_Z)^2$$  \hspace{1cm} (10.01)
where \( i \), refers to the specific bin. This is illustrated in Figure 10.01. The first term on the right hand side of equation 10.01 corresponds to the leading order (LO) contributions to the cross section calculation and the second term corresponds to the next-to-leading order (NLO) terms.

\[
\frac{d\sigma}{dX} = \alpha_s(M_Z) \]

The statistical uncertainty in \( \frac{d\sigma}{dX} \) is propagated using the same equation to give a statistical uncertainty in \( \alpha_s(M_Z) \).

(iii) Then for each bin, \( i \), \( \frac{d\sigma}{dX} = \alpha_s(M_Z) \) is set to the experimental value in equation 10.01 which is then solved for \( \alpha_s(M_Z) \). The statistical uncertainty in \( \frac{d\sigma}{dX} \) is propagated using the same equation to give a statistical uncertainty in \( \alpha_s(M_Z) \).
uncertainty in the corresponding $\alpha_s(M_z)$. This is also illustrated in Figure 10.01. Figures 10.02 and 10.03 show the fits for the $Q^2$ and $E_{T,\text{jet}}^B$ bins from the inclusive jets results and Figure 10.04 shows the corresponding plot for the ratio of inclusive events to dijet events from the ratio analysis.

(iv) Once obtained, the range of $\alpha_s(M_z)$ values can be combined into overall values by means of a $\chi^2$-statistical fit, with the overall statistical uncertainty derived from the fit. For the inclusive jets analysis, four combined values were obtained in this manner; for $Q^2 > 125 \text{ GeV}^2$, $Q^2 > 500 \text{ GeV}^2$, $E_{T,\text{jet}}^B > 8 \text{ GeV}$ and $E_{T,\text{jet}}^B > 14 \text{ GeV}$. It is the second of these values, $Q^2 > 500 \text{ GeV}^2$, that shows the lower combined experimental and theoretical uncertainties and so this is the one quoted as the final result. For the ratio analysis, two combined values were obtained corresponding to $Q^2 > 125 \text{ GeV}^2$ and $Q^2 > 500 \text{ GeV}^2$ and it is the second of these that is quoted as the final result, again because the overall uncertainties are smaller.

The results of the extraction of the $\alpha_s(M_z)$ values (and the combined overall values) with the corresponding uncertainties are summarized in Tables 10.01 and 10.02 for the inclusive jets analysis and Tables 10.03 and 10.04 for the ratio analysis. The systematic and theoretical uncertainties quoted in these tables correspond to those associated with the data points and theoretical calculations, and are described in Sections 9.1 and 9.2.
**Figure 10.02:** Extraction of $\alpha_s(M_Z)$ from $Q^2$ bins.
The black points represent the NLO theoretical points for the corresponding value of $\alpha_s(M_Z)$. The solid line represents the fit. The red points (left) represents the data result. The blue point (bottom) represents the extracted value of $\alpha_s(M_Z)$.

**Figure 10.03:** Extraction of $\alpha_s(M_Z)$ from $E_{T,jet}^B$ bins. Details as in Figure 10.01.
Table 10.01: Summary of extracted values of $\alpha_s(M_Z)$ and associated uncertainties for inclusive jets analysis obtained using the MRST99 PDF set. The main overall value with lowest overall uncertainties is shown in bold.

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Figure 10.04: Extraction of $\alpha_s(M_Z)$ from $Q^2$ bins using the ratio of inclusive events to dijet events. Details as in Figure 10.01.
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<th>Variable</th>
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<th>Central $\alpha_s(M_z)$</th>
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<td>[18,25]</td>
<td>0.1146</td>
<td>0.0029</td>
</tr>
<tr>
<td></td>
<td>[25,35]</td>
<td>0.1188</td>
<td>0.0072</td>
</tr>
<tr>
<td></td>
<td>[35,100]</td>
<td>0.1377</td>
<td>0.0130</td>
</tr>
<tr>
<td></td>
<td>$&gt; 14$</td>
<td>0.1208</td>
<td>0.0015</td>
</tr>
<tr>
<td></td>
<td>$&gt; 8$</td>
<td>0.1200</td>
<td>0.0006</td>
</tr>
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</table>

Table 10.02: Summary of extracted values of $\alpha_s(M_z)$ and associated uncertainties for inclusive jets analysis obtained using the CTEQ4 PDF set.

<table>
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<tr>
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<tbody>
<tr>
<td>$R_{2+1}$</td>
<td>[125,250]</td>
<td>0.1191</td>
<td>0.0010</td>
<td>0.0075</td>
<td>0.0074</td>
<td>0.0098</td>
<td>0.0123</td>
</tr>
<tr>
<td></td>
<td>[250,500]</td>
<td>0.1165</td>
<td>0.0013</td>
<td>0.0059</td>
<td>0.0065</td>
<td>0.0066</td>
<td>0.0089</td>
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<tr>
<td></td>
<td>[500,1000]</td>
<td>0.1163</td>
<td>0.0016</td>
<td>0.0077</td>
<td>0.0078</td>
<td>0.0047</td>
<td>0.0051</td>
</tr>
<tr>
<td></td>
<td>[1000,2000]</td>
<td>0.1174</td>
<td>0.0022</td>
<td>0.0038</td>
<td>0.0038</td>
<td>0.0034</td>
<td>0.0045</td>
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<tr>
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<td>[2000,5000]</td>
<td>0.1202</td>
<td>0.0031</td>
<td>0.0050</td>
<td>0.0047</td>
<td>0.0027</td>
<td>0.0039</td>
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<td>$&gt; 5000$</td>
<td>0.1213</td>
<td>0.0076</td>
<td>0.0085</td>
<td>0.0048</td>
<td>0.0051</td>
<td>0.0025</td>
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<tr>
<td></td>
<td>$&gt; 500$</td>
<td><strong>0.1173</strong></td>
<td><strong>0.0012</strong></td>
<td><strong>0.0027</strong></td>
<td><strong>0.0026</strong></td>
<td><strong>0.0043</strong></td>
<td><strong>0.0048</strong></td>
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<td>$&gt; 125$</td>
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<td>0.0041</td>
<td>0.0043</td>
<td>0.0073</td>
<td>0.0085</td>
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Table 10.03: Summary of extracted values of $\alpha_s(M_z)$ and associated uncertainties for ratio analysis obtained using the MRST99 PDF set.

The main overall value with lowest overall uncertainties is shown in bold.
10.2 Running of $\alpha_s(M_Z)$

Figure 9.01(a) shows six points representing the measured differential cross section at six values of $Q^2$. For each of these points a value of $\alpha_s(M_Z)$ was obtained by calculating the appropriate cross section using a range of assumed values of $\alpha_s(M_Z)$ and extracting a value of $\alpha_s(M_Z)$ for which the theoretical calculation of the cross section matches the experimental value (as described in Section 10.1). These $\alpha_s$ values were extracted at a scale of $M_Z$. However, not all these points in Figure 9.01(a) correspond to a value of $Q^2$ which is equivalent to this scale. The lowest point corresponds to a value of $Q = 13.28$ GeV. As a consistency check, the Renormalization Group Equation can be used to scale the values of $\alpha_s(M_Z)$ to any other scale. So for the lowest point in Figure 9.01(a), a value of $\alpha_s$ can be obtained at a scale of 13.28 GeV, which corresponds to the scale at which the measurement was made. This was done for each point in the cross section plots used in extracting $\alpha_s$ values. This variation of $\alpha_s$ with scale is known as the
running of $\alpha_s$. Table 10.05 shows values of $\alpha_s$ extracted at the scales corresponding to that at which the cross section measurement is made, with associated uncertainties, for the inclusive jets analysis. Table 10.06 shows the equivalent results for the ratio analysis. These values were then compared to those predicted using the Renormalization Group Equation starting with a value of $\alpha_s(M_Z)$ equal to the world average of 0.1182 ± 0.0027. Figure 10.05 shows the running of $\alpha_s$ with respect to $Q$ for the inclusive jet analysis and Figure 10.06 shows the same running of $\alpha_s$ with respect to $E_{T,\text{jet}}^B$. Figure 10.07 shows the running of $\alpha_s$ with respect to $Q$ for the ratio analysis. In each of these plots the solid line represents the running of $\alpha_s$ calculated using the Renormalization Group Equation based on a value of $\alpha_s(M_Z)$ equal to the world average.

<table>
<thead>
<tr>
<th>Variable, $X$</th>
<th>$&lt;X&gt;$ (GeV)</th>
<th>$\alpha_s(&lt;X&gt;)$</th>
<th>Statistical Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>13.28</td>
<td>0.17664</td>
<td>0.00163</td>
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<td></td>
<td>18.66</td>
<td>0.15674</td>
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<td>26.33</td>
<td>0.14773</td>
<td>0.00164</td>
</tr>
<tr>
<td></td>
<td>37.20</td>
<td>0.13896</td>
<td>0.00183</td>
</tr>
<tr>
<td></td>
<td>54.00</td>
<td>0.12942</td>
<td>0.00212</td>
</tr>
<tr>
<td></td>
<td>84.31</td>
<td>0.12583</td>
<td>0.00425</td>
</tr>
<tr>
<td>$E_{T,\text{jet}}^B$</td>
<td>8.92</td>
<td>0.18822</td>
<td>0.00237</td>
</tr>
<tr>
<td></td>
<td>11.67</td>
<td>0.17376</td>
<td>0.00162</td>
</tr>
<tr>
<td></td>
<td>15.96</td>
<td>0.16647</td>
<td>0.00193</td>
</tr>
<tr>
<td></td>
<td>20.65</td>
<td>0.15124</td>
<td>0.00206</td>
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<tr>
<td></td>
<td>28.49</td>
<td>0.14781</td>
<td>0.00374</td>
</tr>
<tr>
<td></td>
<td>41.26</td>
<td>0.15278</td>
<td>0.01276</td>
</tr>
</tbody>
</table>

Table 10.05: Summary of extracted values of $\alpha_s(<X>)$ and statistical uncertainties for inclusive jets analysis obtained using the MRST99 PDF set.
### Table 10.06: Summary of extracted values of $\alpha_s(<Q>)$ and statistical uncertainties for ratio analysis obtained using the MRST99 PDF set.

<table>
<thead>
<tr>
<th>Variable, $Q$</th>
<th>$&lt;Q&gt;$ (GeV)</th>
<th>$\alpha_s(&lt;Q&gt;)$</th>
<th>Statistical Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>13.19</td>
<td>0.16921</td>
<td>0.00207</td>
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<tr>
<td></td>
<td>18.93</td>
<td>0.15288</td>
<td>0.00228</td>
</tr>
<tr>
<td></td>
<td>26.81</td>
<td>0.14282</td>
<td>0.00245</td>
</tr>
<tr>
<td></td>
<td>35.91</td>
<td>0.13570</td>
<td>0.00297</td>
</tr>
<tr>
<td></td>
<td>55.97</td>
<td>0.13073</td>
<td>0.00369</td>
</tr>
<tr>
<td></td>
<td>83.12</td>
<td>0.12285</td>
<td>0.00780</td>
</tr>
</tbody>
</table>

### Figure 10.05: Running of $\alpha_s$ with respect to $Q$ for inclusive jets analysis. Black points represent data measurements. Inner bars represent the statistical uncertainties. Outer bars represent the total experimental uncertainties. Dotted line bars represent the theoretical uncertainties.
Figure 10.06: Running of $\alpha_s$ with respect to $E_{T,jet}$ for inclusive jets analysis. Details are given in Figure 10.05.

Figure 10.07: Running of $\alpha_s$ with respect to $Q$ for ratio analysis. Details are given in Figure 10.05.
The plots in Figures 10.05 to 10.07 show that the values of $\alpha_s$ extracted from the data are consistent with the theoretical curves produced using the Renormalization Group Equation corresponding to a value of $\alpha_s(M_Z)$ equal to the world average. This consistency, over a wide range of scales, is further evidence of the validity of the QCD theory used in the calculations.
11 Conclusions and Outlook

11.1 Summary

Two analyses have been presented in this thesis. Each one has the aim of testing pQCD theoretical predictions and obtaining a value of the QCD parameter, $\alpha_s(M_Z)$, using high $E_T$ jets from DIS events of virtuality, $Q^2 > 125 \text{ GeV}^2$. The data were collected by the ZEUS detector at HERA during the running period 1998–2000 and consisted of e$^-$p and e$^+$p collision events. Theoretical predictions are calculated to NLO in $\alpha_s$ using the DISENT program. This generates configurations corresponding to the Feynman diagrams for the relevant processes in the perturbative expansion, for partons in the initial and final states, each with an associated weight. Each of these configurations is convoluted with the proton PDF. The MRST99 proton PDF was used for the main analysis with the CTEQ4 proton PDF used as a cross check. DISENT calculates matrix elements internally and removes IR divergences from the calculation using the Subtraction Method and the Dipole Formalism.

The first analysis measured differential cross sections with respect to $Q^2$ of the event, $E^B_{T,\text{jet}}$ and $\eta^B_{\text{jet}}$ for inclusive jets, i.e., all jets in the event surviving the cuts, $E^B_{T,\text{jet}} > 8 \text{ GeV}$ and $-2.0 < \eta^B_{\text{jet}} < 1.5$. The NLO calculations agree to within the total uncertainties with the data measurements for all three differential cross sections, which confirms the validity of the theoretical calculations. The value of $\alpha_s(M_Z)$ was extracted
from a QCD fit of the differential cross sections with respect to $Q^2$ and $E_{T,jet}^B$. The values with the lowest overall uncertainties were

$$\alpha_s(M_Z) = 0.1196 \pm 0.0010 \text{ (stat.)} + 0.00020 \text{ (syst.)} + 0.00034 \text{ (th.)} - 0.00025 \text{ (syst.)} - 0.00014 \text{ (th.)} \quad \text{for } Q^2 > 500 \text{ GeV}^2,$$

$$\alpha_s(M_Z) = 0.1198 \pm 0.0008 \text{ (stat.)} + 0.00035 \text{ (syst.)} + 0.00033 \text{ (th.)} - 0.00030 \text{ (syst.)} - 0.00021 \text{ (th.)} \quad \text{for } E_{T,jet}^B > 14 \text{ GeV}.$$

The second analysis measured cross sections with respect to $Q^2$ and $E_{T,jet}^B$ for exclusive dijet events with $E_{T,jet1}^B > 8 \text{ GeV, } E_{T,jet2}^B > 5 \text{ GeV}$ and $-2.0 < \eta_{jet}^B < 1.5$. Again, the NLO calculations agree to within the total uncertainties with the data measurements for both these differential cross sections, which confirms the validity of the theoretical calculations. The differential cross section with respect to $Q^2$ for inclusive DIS events was also measured. The ratio of the exclusive dijet events to the inclusive DIS events with respect to $Q^2$ was obtained, and consistency is shown to within the uncertainties, between the data measurement and the NLO calculation. This ratio was used to obtain $\alpha_s(M_Z)$ from a QCD fit, in the same way as was done for the inclusive jet measurements. The value of $\alpha_s(M_Z)$ obtained was

$$\alpha_s(M_Z) = 0.1173 \pm 0.0012 \text{ (stat.)} + 0.00026 \text{ (syst.)} + 0.00048 \text{ (th.)} - 0.00027 \text{ (syst.)} - 0.00043 \text{ (th.)} \quad \text{for } Q^2 > 500 \text{ GeV}^2.$$

These values are consistent with other values of $\alpha_s(M_Z)$ obtained in other analyses at HERA, as well as the world average ($\alpha_s(M_Z) = 0.1182 \pm 0.0027$)[10]. This is illustrated in Figure 11.01. It can be seen that the values obtained in these analyses compare well with
other values obtained at HERA, as well as the world average, but with significantly improved experimental and theoretical uncertainties over previous results.

**Figure 11.10:** Measurements of $\alpha_s(M_Z)$ made by analyses with data obtained at HERA. Details can be found in [70-80]. Points (m), (n) and (o) represent results from the analyses described in this thesis (**). The shaded band represents the world average with uncertainties.
The running of $\alpha_s$ was also checked as a function of $Q^2$ and $E_T$ and agreement between the data measurements and the theoretical calculation was also seen thereby validating the QCD theory used in those calculations.

11.1.1 Analysis Improvements

The main improvement to these analyses would lie in the reduction of the uncertainties involved. On the experimental side, the largest uncertainty arises from the uncertainty in the jet energy scale. The only way to improve this would be an improvement in the understanding and operation of the detector.

Since obtaining the data for this analysis, both HERA and ZEUS have undergone a significant upgrade. HERA is now running with much higher beam intensities, which allow a much higher integrated luminosity to be acquired (i.e., more events in a shorter time). To deal with the increase in instantaneous luminosity, the ZEUS detector has had an upgrade in the trigger system, electronics and software. Also, new detector components have been installed, in particular the MicroVertex Detector (MVD) and the Straw Tube Tracker (STT). The MVD is a silicon based tracking detector encompassing the interaction point at the centre of the detector. It provides much higher precision in locating the primary and secondary vertices from the decay of particles coming from the interaction. The STT provides a large improvement in tracking in the forward direction and complements that already provided by the CTD.
These improvements should yield a data set with much higher statistics, which should reduce the statistical and systematic uncertainties, as well as improved tracking and reconstruction of the scattered electron and jets, particularly in the forward direction.

On the theoretical side, the largest uncertainty is due to the choice of renormalization scale, $\mu_R$. This uncertainty is estimated by varying the choice of renormalization scale by a factor of two each way and is designed to estimate the uncertainty due to cutting off the perturbation series used in calculating the theoretical predictions at next-to-leading order. One way of reducing this uncertainty would be to include higher order terms in the calculation, (i.e., perform calculations at NNLO). So far these calculations are not yet available. The analyses are carried out in a phase space where this particular uncertainty is minimized, i.e., at high $Q^2$ and $E_{T,jet}$. Should higher order calculations become available then these cuts could be lowered.

11.2 Further Analysis: Jets with Charmed Mesons

11.2.1 Introduction

The requirement of a charm quark in the event adds another scale to the hard process. This analysis is an extension of the dijet analysis. This section briefly describes how that analysis would be done and shows some of the preliminary results.
11.2.2 Selection of Events

The additional requirement of a charm quark drastically reduces the available statistics for this analysis\(^\text{17}\). For this reason it is necessary to use the widest possible cuts. For the selection of DIS events, the same cuts are used as for the ratio analysis except that the \(Q^2\) cut is reduced to 4.0 GeV\(^2\). In addition to the DIS selection, the events were required to have exactly two jets, one of which is tagged with a \(D^*\) meson candidate. Candidates for \(D^*\) are reconstructed using the so-called Golden Decay channel,

\[
D^{*\pm} \to D^0 \pi^\pm \to K^\mp \pi^\pm \pi^\pm.
\]

The following requirements were placed on \(D^*\) candidates and their decay products:

- \(p_T(D^*) > 3.0\) GeV
- \(p_T(K,\pi) > 0.4\) GeV
- \(p_T(\pi) > 0.15\) GeV, the so-called ‘slow pion’ due to the limited phase space available.
- \(|\eta_{\text{track}}| < 1.5\), i.e., decay product tracks are seen in the CTD.
- \(1.80 < m(D^0) < 1.92\) GeV.
- \(0.143 < \Delta M < 0.148\) GeV, where \(\Delta M = m(D^*) - m(D^0)\).

Using this quantity leads to large suppression of the combinatorial background.

These cuts are standard in the selection of \(D^*\) candidates at ZEUS and are used in numerous charm related analyses. Examples include those described in [81]. The

\(^{17}\) It is for this reason, the enhanced statistics of HERA II will be very useful to this analysis.
combinatorial background was modeled using wrong sign combinations of the appropriate decay products.

Jets are reconstructed in the laboratory frame using the $k_T$-cluster algorithm in inclusive mode and are subject to the following selection criteria:

- $E_{T,jet1}^L > 6.0$ GeV and $E_{T,jet2}^L > 5.0$ GeV.
- $0.5 < \eta_{jet} < 2.4$.
- As with the main analyses, events with a jet close to the scattered electron are completely rejected.

The asymmetric jet energy cut is used to avoid problems in the NLO calculation as explained in the ratio analysis. The cuts are slightly lower due to the limited statistics involved.

In the data jets are matched to a $D^*$ candidate if the following condition is satisfied:

$$\sqrt{(\eta_{D^*} - \eta_{jet})^2 + (\phi_{D^*} - \phi_{jet})^2} < 0.6$$

The Monte Carlo generator used was RAPGAP. The resulting output was processed by the same detector simulation software as the main analyses. The Monte Carlo has all the generator level information available and so at the hadron level, the $D^*$'s can be directly assigned to the jets by testing which particles are used to make up the jet.

### 11.2.3 Control Plots

The control plots for dijet events containing a matched $D^*$ are shown in Figures 11.02 to 11.04 below. Plots are shown for variables involved in the selection of events, $D^*$'s and
jets. Due to the limited statistics involved in this analysis, the error bars on these plots are much larger. For the event control plots there appears to be some discrepancy in the agreement between the data and MC, in particular for the $Q^2$ plot. However, it can be seen that there is agreement at the two sigma level for all these plots.

For the control plots relating to the $D^*$ and its decay products it can be seen that the agreement between the data and MC is also well within the two sigma level. The uncertainties for the jet variables shown are much smaller due to the higher number of jets, and consequently the agreement between the data and MC is much better.

In conclusion, the agreement shown by these control plots is considered to be sufficient to consider the RAPGAP MC suitable for this analysis.
Figure 11.03: \(D^*\) control plots for charm dijet analysis. Plots (a) and (b) show variables, \(P_T\) and \(\eta\), relating to the reconstructed \(D^*\). Plots (c) to (h) the same variables for the decay products of the \(D^*\) (i.e. the K, \(\pi\) and \(\pi_s\)).
11.2.4 Signal

Once all these conditions have been applied, a mass difference, $\Delta M = M_{D^*} - M_{D^0}$, distribution can be obtained. This is shown in Figure 11.05. The advantage of this distribution lies in the relative masses of the decay products in the Golden Decay channel. The difference in the rest masses of the $D^*$ and $D^0$ is very close to the rest mass of a pion. So when a $D^*$ decays through this channel to form a $D^0$ and a pion, there is very little energy for the kinetic energy of the pion. For this reason the pion is referred to as the ‘slow’ pion, $\pi_s$. By plotting the mass difference distribution the background products are highly suppressed due to this lack of phase space for the initial decay. Hence a clear signal can be seen in Figure 11.05.

**Figure 11.04:** Jet control plots for charm dijet analysis.
A modified Gaussian was used to fit the signal plus background and establish the number of matched $D^*$-jets. There are a number of possibilities for the form of this fit that can be justified. The one used is given in Equation 11.01.

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot \frac{0.0054 A_1}{A_2} \cdot \exp \left[ \frac{(x - A_6)^2}{2A_2^2} \right] + A_3 (x - m_\pi)^{\lambda_4} \cdot \exp[A_5 (x - m_\pi)] \quad x \geq m_\pi$$

$$= 0 \quad x < m_\pi \quad (11.01)$$
where $A_i$ ($i = 1..6$) are the unknown fit parameters and $m_\pi$ is the mass of the pion. The first term on the right hand side of equation 11.01 represents the Gaussian peak of the signal and the second term represents the background distribution and consists of a power term multiplied by an exponential term that represents the shape of the background. The discontinuity at $\Delta M = m_\pi = 0.139$ GeV arises because no signal can be seen below this threshold. From this fit the number of $D^*$-jet pairs was found to be $723 \pm 30$ from a total integrated luminosity of $120.1$ pb$^{-1}$.

### 11.2.5 Corrections Factors

As with the main analyses the differential cross sections measured need to be corrected for detector effects. These acceptance corrections incorporate, not only effects due to the detector efficiency and geometry, but also the trigger efficiency and the purity and efficiency of the jet finding and $D^*$ reconstruction. The differential cross sections measured and presented here are with respect to $Q^2$, $E^{T,jet}_T$, $\eta^T_{jet}$, $P_{T,D^*}$, $\eta_{D^*}$ and $\theta^*$ (the angle between the two jets, charm and non-charm). The acceptance correction factors are shown in Figure 11.06. It can be seen that these correction factors are all between two and three which is somewhat higher than for the main analyses. This reflects the added inefficiency in reconstructing the $D^*$ particles used to tag the charm jets. However these acceptance factors are consistent with previous analyses done at HERA[81] and thought to be sufficiently under control for the analysis to be performed.
Figure 11.06: Acceptance correction factors for charm dijet cross sections.

Figure 11.07: Charm tagged dijet event cross sections.
11.2.6 Cross Sections

The cross section plots are presented in Figure 11.07. The cross sections are shown with respect to $Q^2$, $E_{T,\text{jet}}$, $\eta_{\text{jet}}$, $P_{T,D^*}$, $\eta_{D^*}$, and $\theta^*$. Due to limited statistics only four points are possible in for each plot.

11.2.7 Summary and Potential Future Work

The analysis presented in this section is an example of the further work that can be carried out in this field. Only the bare cross section measurements are presented albeit for relatively low statistics. To continue this work to its logical conclusion, a larger statistical sample would be required, which has now become available at ZEUS since the upgrade at DESY to HERA II. To get a value of $\alpha_s(M_Z)$ using the same methods as the main analyses requires the use of appropriate NLO calculations, which are also now becoming available. These calculations are much more complicated than the calculation of inclusive jet and dijet cross sections due to the extra hard scale introduced by the presence of the charm quark, hence it is has not yet been possible to perform them. The work described in this section should provide the basis for a much more thorough analysis involving charm tagged jets in deep inelastic scattering events that will provide a further test of pQCD and allow for a new method of extracting the QCD parameter $\alpha_s$ in the future.
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